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ABSTRACT

Mathematics and the use of mathematical thinking should be much more than what has been traditional school arithmetic. Much of the mathematical reasoning can be developed and experienced out of school, particularly in the home. This material is a teacher's guide designed to help parents support what is done with their children in class. Background material for parents is provided. Some underlying principles in teaching mathematics meaningfully are listed. Assessment record sheets are presented. A total of 58 activities on the following concepts and skills are included: (1) comparing; (2) counting; (3) classification; (4) using patterns; (5) number; (6) number operations; (7) problem solving; (8) place value; (9) equality; (10) fractions; (11) correspondences; (12) geometry; (13) logic; (14) estimation; and (15) measurement. (YP)

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MEANINGFUL MATHEMATICS

LEVEL TWO

TEACHER'S GUIDE TO LESSON PLANS

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LEVEL TWO
USING THE TEACHERS' GUIDE

The lessons are grouped by topic - counting, place value, etc. Develop the concepts for mastery: Classification, Patterns, Number, Addition, Subtraction, Multiplication and Division, Place Value, Numeration, Using the Operations in Problems, Fractions, Geometry, etc. Then repeat lessons or use more advanced lessons in any order to provide repetition and practice. No specific scope and sequence is provided because mastery of foundation ideas is the major goal of this level.

LEVEL TWO
INTRODUCTION

Coming into this level students will have had work with:

1. Number concept development
2. Meaning and use of the four arithmetic operations
3. A basis for place value representation
4. Problem solving
5. Recognition of 3D and 2D geometric shapes
6. Logic
7. Graphing
8. Measurement

All of this has been introductory, and, in order to continue with the children, you must do some assessment of this development prior to starting.

While emphasis is on mastery of each concept, it is a good idea to periodically build in an activity that reviews all concepts and skills previously learned. Once each week is a good schedule for elementary school. This can be done orally in the primary grades and with a written exercise after symbolic representations have been mastered. Since all operations arise from combining, separating, comparing and part whole relations, as soon as the operation symbols are understood, these operations should all be presented at the same time in the written review exercises.

It is easier to do this at the beginning levels with orally presented review activities.

It is axiomatic for good teaching that past learnings should ALWAYS be integrated into new learning activities.

LEVEL TWO

GETTING STARTED

READ the **Level One Teacher's Guide,** and **Mathematics Their Way** as background and for information about what children have done before. Also read **The Piaget Primer,** or a similar work to understand the theory underlying the program.

MAJOR IDEAS AND PROCESSES emphasized during the year are COMPARING, COUNTING, CLASSIFICATION, USING PATTERNS, NUMBER, NUMBER OPERATIONS, PROBLEM SOLVING, PLACE VALUE, EQUALITY, FRACTIONS, CORRESPONDENCES, GEOMETRY, LOGIC, ESTIMATION and MEASUREMENT.

COMPARING will involve comparison of weight, quantity, volumes, areas, parts of wholes, number properties of collections and properties of shapes.

COUNTING will include counting activity for 5 minutes or so each day. Counting, counting on, counting back, skip counting, decade counting, and counting by hundreds are all needed. This should be integrated with estimation by having students estimate amounts prior to counting or calculation. At the end of this level, children should count fairly large amounts accurately, and use the counting number sequence words.

CLASSIFICATION will be related to the language of logic - AND, OR, NOT - and will emphasize similarities and differences in objects in terms of their properties.

USING PATTERNS will emphasize number patterns and relationship patterns and will involve patterns of color, sound and shape less than in LEVEL ONE.

NUMBER CONCEPT should build in a systematic way upon number concept already developed in prior work. Assessment of children's concept of number is imperative at the beginning of the year. Other activities to develop place value, equality and problem solving will help reinforce number concept at numeration. NUMBER STATIONS should be set up at the beginning of the year. Assessment should determine which number each child should start with. Stations using UNIFIX CUBES at one, TILES at another, COLORED BEANS at another, etc., should be used. NUMBER STATION work should help children internalize the 2, 3 and 4 part combinations and equal part combinations of the different numbers.

PLACE VALUE work should begin with a repeat of the counting on and counting back activities as done in LEVEL ONE with group sizes smaller than ten. However, the emphasis should shift to TEN when all children have mastered "Tenness." Use Unifix cubes to start with and then gradually introduce "exchanges" by trading ONES for TENS in base ten form or for the orange Cuisenaire Rod. Children should look for FIVE combinations and "see" seven as five & two, nine as five & four, etc. The best way to do this is by using TEN frames on recording forms and with overhead transparency activities or using bean sticks with beans glued on in two rows of five.

The usual counting up games and counting back games can be used to emphasize Base Ten place value and numeration activites.

ARITHMETIC OPERATIONS - Children will still need separating, joining and comparing activities with both unequal parts and equal parts. Unifix cubes and counters can be used to separate a large collection into (1) 2 or more unequal parts in different ways (2) as many parts as possible for a given smaller size, (3) into equal parts of a given number. These also can be used to join together (1) wo or more unequal parts and (2) a given number of parts of the same size. Comparisons of 2 unequal groups should be made during both a difference comparison and a ratio comparison.

The meaning of addition, subtraction, multiplication and division should all be firmly established by the end of the year through these activities. The addition and subtraction will be integrated with place value in Base Ten through the development of these computations in Base Ten. The computation algorithms for multiplication and division will be in later levels, but must be based in a sound understanding of what these mean in LEVEL TWO.

PROBLEM SOLVING will involve word problems requiring addition, subtraction, multiplication and division. This is reviewed with small numbers < ten and continued with larger numbers. More two step problems are to be used. Non-numeric problem solving using pattern blocks, tangrams and geoboards should be emphasized to a greater degree.

Relate problems to children's experience. Children should write problems and create problems for others to solve. To promote problem solving, arrange challenges with a small gap between the demands of the situation and application of prior knowledge and skills. Accept whatever problem solving strategies are used, but encourage each to attempt other ways to solve problems. Problem solving is the primary goal of this program!

EQUALITY activity is to be extended to the use of Base Ten materials. The emphasis should still be on the ideas of "less than", "more than", and "same as", as well as generating number sentences of the forms: $a + b + \ldots k = x$; $x = a + b + \ldots + k$, and $a_1 + b_1 + \ldots k_1 = a_2 + b_2 + \ldots + k_2$.

FRACTION concept development activity will be extended to separating, joining, and comparing fractional parts in preparation for formal work with computation with fractions.

MENTAL COMPUTATION will be emphasized along with calculator activity. Children should develop immediate recall of "addition" facts and "multiplication" facts for smaller numbers during the year.

GEOMETRY will continue to be emphasized to a greater degree with activity involving two dimensional and three dimensional shapes. Geoblocks, Geoboards, Tangrams and Pattern Block activities are in large number.

CORRESPONDENCES other than 1-1 correspondence are introduced to lay groundwork for understanding of ratio. Arrangements of Unifix Cubes will lead to symbolizing correspondences like: 4:3, 2:3; and correspondence classes like 2:3, 4:6, 6:9, 8:12, etc.

ESTIMATION should precede any counting, computation or measuring.

MEASURING will involve both non-standard and standard units of measure of length, area and volume. Using numbers to represent the results of measuring must be introduced only as children indicate readiness for it. Measuring can be integrated with place value and other activities.

MATERIALS USED during the year include:

Unifix cubes
Base ten blocks
Place value mats
Pattern blocks
Tangrams
Geoblocks
Logic blocks (ASCOBLOCKs or Attribute blocks)
Cuisenaire Rods
Geoboards

LEVEL TWO: INSTRUCTIONAL PRACTICES

1. Most lessons are designed so that introduction is by demonstration of how to use the materials on the overhead. Do this and ask enough questions so children know what they are to do. When recording forms or worksheets are involved, make transparencies of these and show the children how to record in them.

2. Introduce any symbols slowly and carefully so that children know what they represent.

3. Group children into 2's, 3's and 4's as often as possible and encourage them to learn together and from each other.

4. Recording forms and worksheets should be kept in individual student folders labelled "My Work in Mathematics."

5. When children are working with materials, walk around and observe what they are doing. Ask questions of a leading or probing nature to stimulate their thinking. Avoid "playing teacher" and telling them how to do something until they have had sufficient time to work it out for themselves.

6. Children learn from their mistakes. Don't penalize them for "good" mistakes that indicate an idea that just hasn't matured yet. Children get right answers for wrong reasons and wrong answers for right reasons. You must be able to distinguish between these.

7. Plan for storage of materials in open locations where they are accessible to children. Use storage containers like plastic ice cream pails, gallon milk bottles and boxes made into drawers. Teach children to be responsible for getting materials out to work with, policing the work areas for dropped materials and for putting materials away. These are LEARNING materials and children must understand that.

8. Feel free to develop homemade "analogues" of the commercial materials suggested.

ASSESSMENT RECORD
for

*Mathematician:*_____

Assessment: "Please count for me." Circle the largest number in the sequence the child uses correctly.

Date:	Number Sequence
	below ten 11 12 13 14 15 16 17 18 19 20 30 40 50 60 70 80 90 100 110 120 _____(write in)
Date:	below ten 11 12 13 14 15 16 17 18 19 20 30 40 50 60 70 80 90 100 110 120 _____(write in)
Date:	below ten 11 12 13 14 15 16 17 18 19 20 30 40 50 60 70 80 90 100 110 120 _____(write in)

*Mathematician:*_____

Assessment: "Please Count These." (rational counting of objects. Circle the largest number in the set the child correctly counts.

	Number Sequence	Moves Objects	Touches Objects	Mentally "Touches"
Date:	below ten 11 12 13 14 15 16 17 18 19 20 30 _____ (write in)			
Date:	below ten 11 12 13 14 15 16 17 18 19 20 30 _____ (write in)			
Date:	below ten 11 12 13 14 15 16 17 18 19 20 30 _____ (write in)			

*Mathematician:*_____

Assessment: Numbers "Known" - all parts combinations.
Circle the largest number on that date.

	Hand Test	Bowl Test	Number
Date:			6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 _____(write in)
Date:			6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 _____(write in)
Date:			6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 _____(write in)

*Mathematician:*_____

Assessment: Writing Counting Sequence in Other Bases. Check the base correctly written to a "full house" (largest 2 digit value.)

	Base								
	Three	Four	Five	Six	Seven	Eight	Nine	Ten	Comment:
Date:									
Date:									
Date:									

*Mathematician:*_____

Assessment: Recognition of Geometric Shapes. Check those recognized or differentiated from others.

	Triangle	Square	Circle	Rectangle	Parallellogram	Hexagon
Date						
Date						
Date						
Date						

LEVEL TWO

COUNTING

Throughout the year the children should have two or three minutes of oral counting every day, supplemented by rational counting of objects such as UNIFIX cubes.

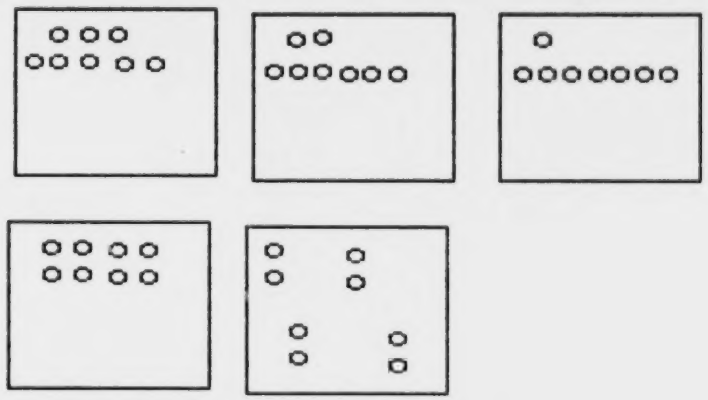
ORAL	RATIONAL
Counting up as high as as possible	Counting out increasingly large collections:
Counting back from a given number	one at a time
Skip counting by twos*	two at a time (count twos obtained)
Skip counting by threes	three at a time (same)
Skip counting by fours	four at a time
Skip counting by fives*	five at a time
Skip counting by sixes	six at a time
Skip counting by sevens	seven at a time
Skip counting by eights	eight at a time
Skip counting by nines	nine at a time
Skip counting by tens*	ten at a time
*particularly important	Count on from a given cube in a long link.
Count off by twos, threes, etc.	Count on starting with the ____th cube
	Find the _____th cube from the left
	Find the _____th cube from the right

LEVEL TWO

NUMBER CONCEPT 1

At number stations, use recording forms with space for 4 addition number sentences for each part-part-whole relation generated. Also include equal part recording forms, i.e. 12 = 3 x 4; 12 = 4 x 3; 3 x 4 = 12; 4 x 3 = 12. Examples of these are attached. Any number concept development activity to be used with a group from LEVEL ONE can be used as seen fit. These supplement the NUMBER STATION work. The lessons given below supplement the LEVEL ONE kinds of lessons and those that can be found on pages 237-240 of Mathematics Their Way. Any of these can be modified or extended upon. Number concept work should continue throughout the year with ever increasing numbers.

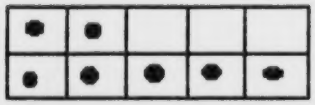
LESSON ONE:
On the overhead projector, place objects, eight to twelve in number so children can see them. They are to write the associated number sentence on the lap chalkboard and hold it up for you to see. Encourage them to write these in as many ways as they can. Some suggested arrays for eight:



Repeat this with a different number for a different lesson.

LESSON TWO:
Choose a number for the day, i.e. eleven. The children are to use UNIFIX cubes and a given number of operation so the end result is eleven. For example: 3 "+" and 2 "-", 5 to start 5 + 8 - 3 + 5 - 6 + 2 = 11.
Vary (1) the starting number, (2) the ending number, and (3) the number and kinds of signs for several new lessons.

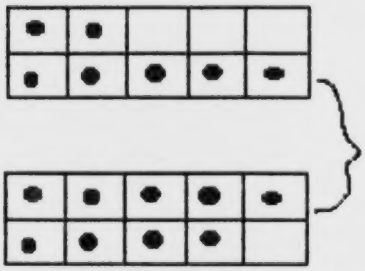
LESSON THREE:
Use the Master provided to make overhead transparency "ten frames". First put these on the overhead one at a time so children look for the total < ten.



IS 7, FOR EXAMPLE

When they can quickly identify all numbers < ten, introduce the ten frames two at a time. Remind them that ten is TWO fives, so they should (1) look for two fives to make a ten, and (2) the extra counters to add to the ten.

Example:



2(5) = 10 + 6 = 16

This is excellent mental computation practice and reinforces work being done with base ten place value.

LESSON FOUR:

Introduction: This is a lesson that can be done several times during the year by using different numbers. Have a number for the day, ie. twelve. Give each child the "number for the day" number of UNIFIX cubes, tiles or other countable. Ask the children to group these into two groups. Write the starts for several number sentences on the board.

12 =	=12
12 =	=12
12 =	=12
⋮	⋮

Ask the children to give you numbers to complete these that show their groupsings: 12 = 9 + 3; 12 = 10 + 2; 7 + 5 = 12, etc. Then ask them to group their objects into three groups and write the same kinds of sentences as before. Press them to find as many different groupings for twelve as they can.

Activity: Pass out paper and have the children try to fill both sides with number sentences for the "number for the day." Suggest they write some with both "+" and "-" signs.

Attached are Masters for recording forms for using beans to learn numbers greater than ten. After assessment shows children have internalized number so that all part-part-whole relationships are known, and the relationship of a number to numbers less than it are known, advance them one number at a time at the number stations. You can also use the general recording form for UNIFIX for this.

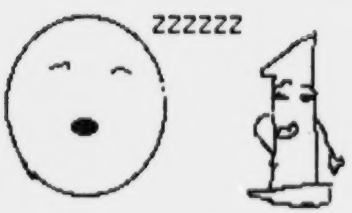
LEVEL TWO

ZERO AND NUMBER CONCEPT

Background: *"The Little Man Who Wasn't There"* has been attributed to Helen MacDonald Simmons and appeared in Mathematics and Humor, a publication of NCTM:

Arithmetic's hero
The fat little zero
He's nothing at all, you say?

Then come with me
I want you to see
Our hero at work and at play!

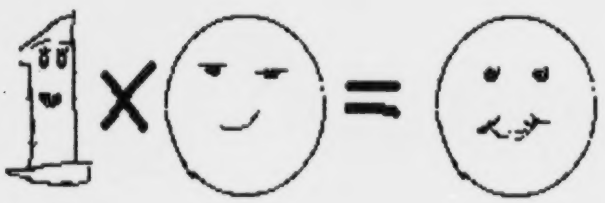


With One, when he flirts
At the left of her skirts
He's done nothing at all, has he.

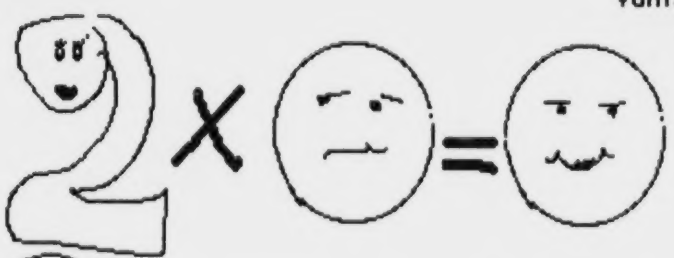
When he stands at her right
He's given her might -
Ten times as great is she.

Add him to One
And nothing is done
Each word that I say is
true.

The same you can say
When you take him away
It's the easiest thing
you can do.

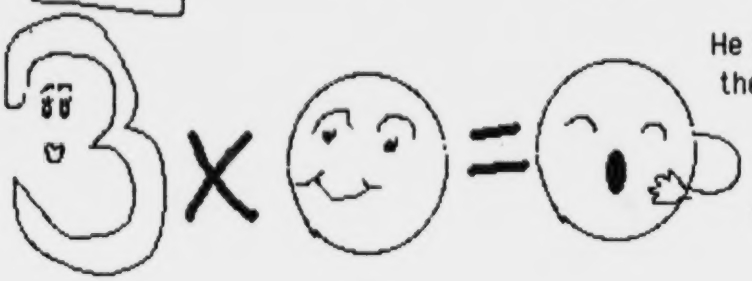


But the damage was
done
When he multiplied
One
She completely
vanished they say.



The same he
would do

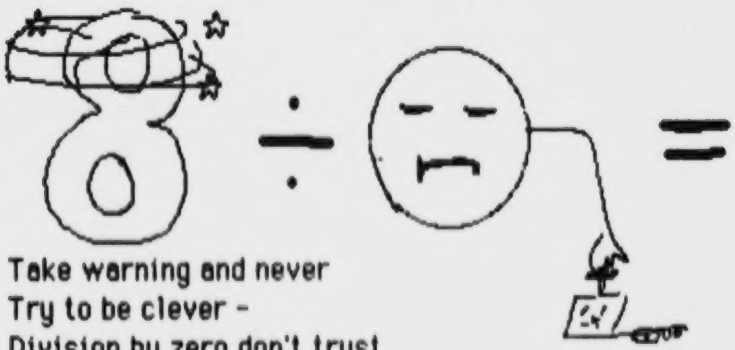
To a Three
or a Two



He treats
them all

the

same way.



Take warning and never
Try to be clever -
Division by zero don't trust.

I'm warning you now
It matters not how
Your answer will blow up and bust!

Emphasize the use of "0", as an additive and subtractive identity, and later its role in multiplication. Be sure that children write the related "0" sentences, when writing the part-part whole sentences for a given number, e.g. for SIX.

6 = 3 + 3	3 + 3 = 6
6 = 2 + 4	2 + 4 = 6
6 = 4 + 2	4 + 2 = 6
6 = 5 + 1	5 + 1 = 6
6 = 1 + 5	1 + 5 = 6
6 = 0 + 6	0 + 6 = 6
6 = 6 + 0	6 + 0 = 6

Following are Masters for recording forms to use at desks or number stations when students are using beans and UNIFIX cubes for part part-whole development of number concept.

PATTERN ACTIVITIES

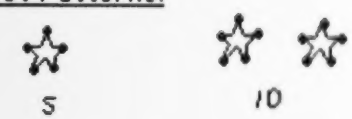
Background: Children will have had considerable experience with copying patterns, making patterns from sound stimuli, and extending patterns in Kindergarten and Level One. At this level patterns should be integrated with other areas - number patterns, place value patterns, counting word patterns, etc.

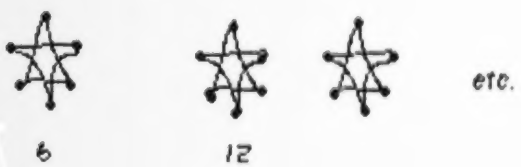
Suggested Pattern Activities: These are over and above those recommended for Kindergarten and Level One. Any of those could be repeated or modified.

Number Patterns:

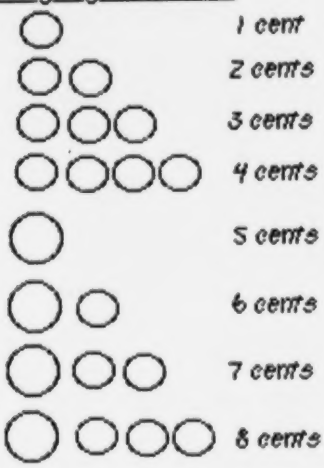
1. 1,3,5,7,9....
2. 1,3,6,10,15...
3. 1,2,4,5,7,8...
4. 2,4,6,8...
5. 3,6,9,12...
6. 5,10,15,20...
7. 4,8,2,16...
8. 6,12,18...

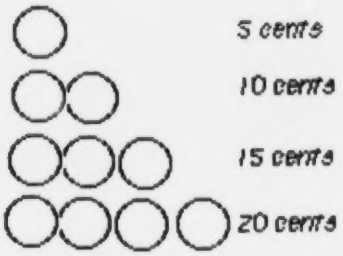
Dot Patterns:





Everyday Patterns:

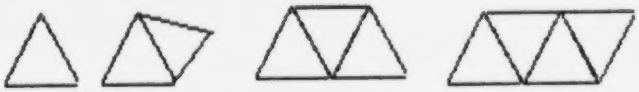




etc.

Toothpick Patterns:

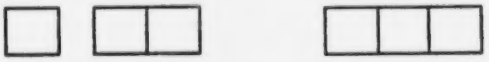
1.



3 5 7 9

etc.

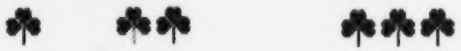
2.



4 7 10

etc.

Clover Leaf, Nut, Leaf Patterns:



3 6 9

etc.

Handshake Patterns:

People	2	3	4	etc.
Handshakes	1	3	6	

(also lines connecting dots)

Tearing Paper in Halves:

tears	0	1	2	3	etc.
pieces	1	2	4	8	

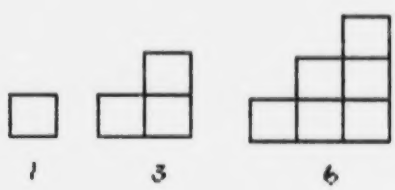
Ordering Patterns:

colors	1	2	3	etc.
orders	1	2	6	

Pattern Block Puzzles:

"Pattern Block Problems for Primary People"
"The Pattern Factory"
Creative Publications

Stair Patterns: (blocks or tiles)



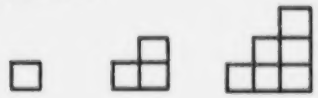
etc.

LEVEL TWO

PATTERNS: TILES

LESSON ONE

Introduction: On the overhead projector, use tiles to begin a number pattern:



Have the children build this pattern with tiles. Ask them to make the next step in the pattern (Prediction). Make the next step:

Ask the children if this is what they made as the next step (Verification). Discuss the pattern and how it is being generated.

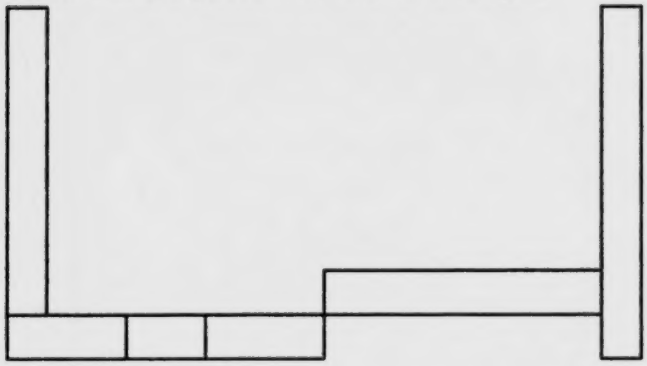
Activity: Have the children make seven elements in this pattern. Monitor their work. When they complete this pass out the related worksheet to do. Ask questions like, "How do you add to each tile shape to get the next one in the pattern?"

LEVEL TWO

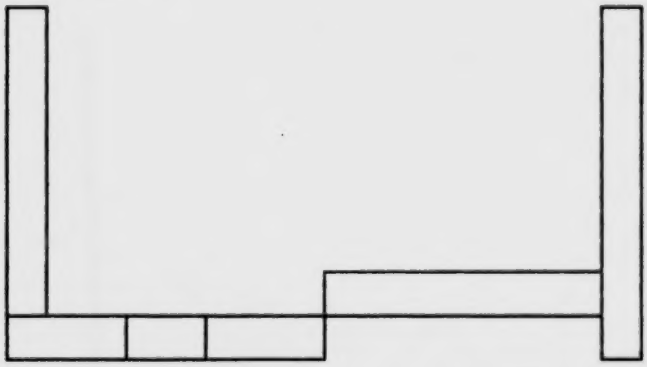
PATTERNS: CUISENAIRE RODS

LESSON ONE

Introduction: Use overhead versions of Cuisenaire Rods - ten or twelve of them. Make a pattern of them. Example:



Point out you used 6 rods. Trace around these to show the outline of the pattern.



Activity: Hand out the recording forms and Cuisenaire Rods. The children are to (1) make a pattern with 5-10 rods
(2) trace around the pattern made on the recording form
(3) print in the number of rods used
Exchange these outline forms so children have the chance to solve several of the resulting rod puzzles.

LEVEL TWO

CLASSIFICATION

Materials to use are any that have recognizably different properties - color, shape, length, size, area, number of holes, textures, etc. If children have put together "Treasure Boxes", these can be. used.

Introduction: This is to give children an experience in distinguishing likenesses and differences in the properties of objects. These may be in the nature of physical properties or properties related to the use of the objects. The important thing to remember is to let the children decide how to sort the objects because of a likeness or difference that they have recognized.

Use two objects and hold them up. You may put them on the overhead projector if they are suitable for that use. Ask the children how they are ALIKE. Discuss these responses. Ask them how they are DIFFERENT. Discuss these responses.

Activity: Have the children take a collection of objects from the treasure boxes or pass out buttons or other material to be used. There should be more than three of each kind such as three similar buttons, three similar keys, etc. The total number including the several kinds should not be more than fifteen or sixteen.

Generally start by sorting into TWO groups, observing which properties are chosen and whether that property is retained until sorting is complete. After much experience sorting into TWO groups using a variety of properties, the sorting should be done into THREE groups.

Questions to ask:

How are the objects in each group alike?
How are the objects in each group different?
What are the differences between groups?

Children of this age should see likenesses and differences that occur one at a time quite easily, for example, the same number of buttonholes, but different shapes. They will not as easily see the joint presence of two or more liknesses, such as the same color AND the same shape.

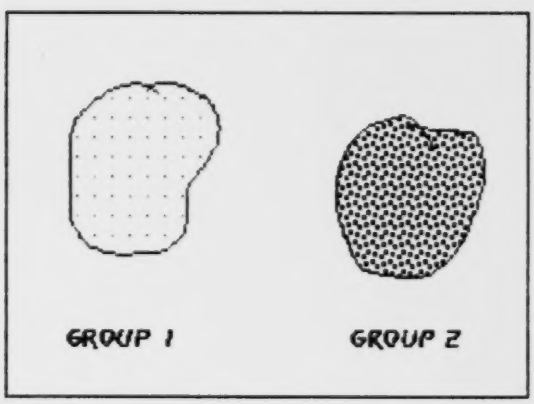
Observe how many can. Do not push those who can't into trying to see this if they do not do it spontaneously. Such multiple classification (AND) develops slowly. Some will do it at this time. Others will as they become ready to after enough classifying experiences.

Use the worksheet and have the children draw in objects that have been sorted into each group. Be sure you orally describe what they are to do. Don't depend on their reading the directions. Spend some time talking about how the things in the groups are ALIKE and how they are DIFFERENT. Have the children describe these liknesses and differences as best they can.

Related Lessons: Classifying experiences of this kind should be done occasionally. A good idea is to have children sort each NEW material when it is first introduced. Use the sequence - two groups, then three groups, or four or five, if that seems appropriate.

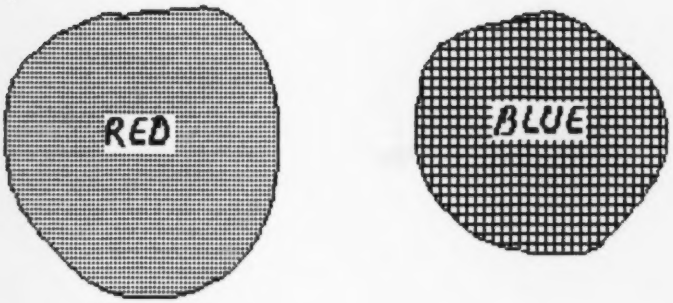
A major goal is to eventually get them to recognize MULTIPLE CLASSIFICATION - the joint presence of two or more properties - red AND square.

During later stages, have children draw the objects put into each group using forms such as:



Spend a lot of time talking about, with the children - SIMILARITIES (ALIKE), DIFFERENCES, AND

Children should also be able to use the either-or meaning of OR



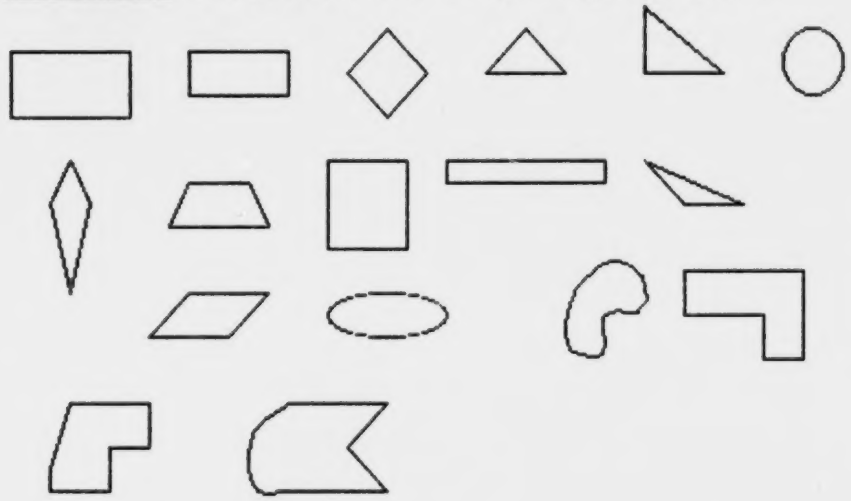
The objects within the boundaries are EITHER red OR blue - none is red AND blue.

Children should classify a collection of objects once every two weeks or so at LEVEL TWO to build upon the work in LEVEL ONE.

LEVEL TWO

CLASSIFICATION: SHAPES

LESSON ONE: Cut out several different shapes as shown:

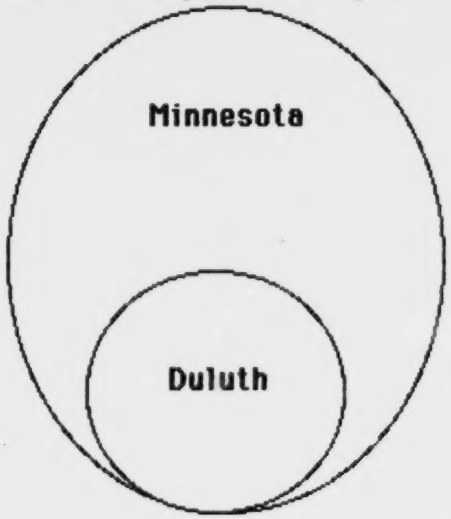


Place these on the overhead. Tell the children you wish to sort them into two groups. "How should I do this?" When the children suggest a way, do it and discuss. Ask why one left out doesn't go into a particular group. "Should this one be in _____ group?" After having discussed this first sorting and pursued the properties that could be used, ask the children for a DIFFERENT way to sort the shapes into two groups and continue as in the first case.

The purpose is to get children to identify the ways in which these common shapes are alike and different and to VERBALIZE these.

CLASSIFICATION: CLASS INCLUSION

LESSON ONE: Make a Venn diagram on the chalkboard or on the overhead as shown: (Use your own city and state)



Questions:

1. Does someone who lives in Duluth live in Minnesota?

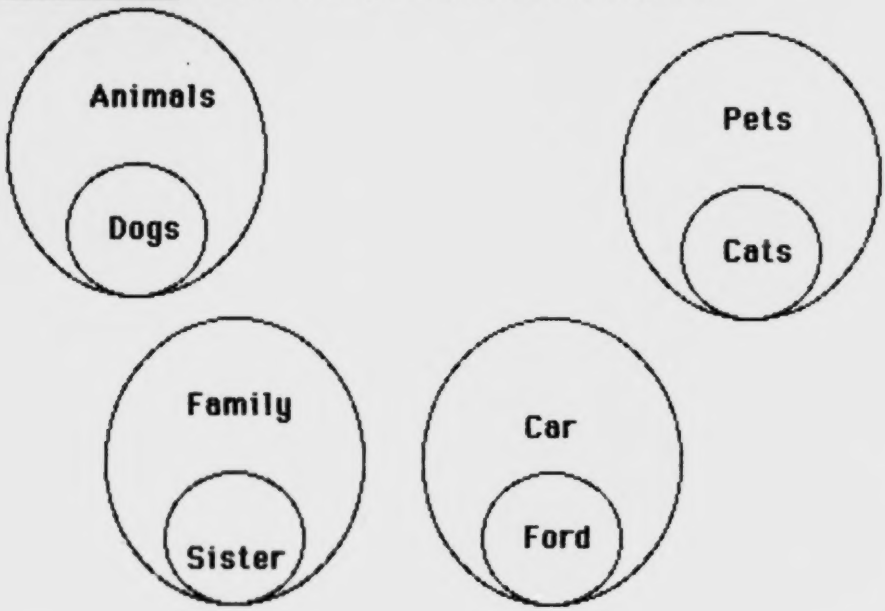
2. Do all people living in Minnesota live in Duluth?

3. Can someone live in Duluth and not live in Minnesota?

4. If I live in Duluth, then _____.
(complete the sentence)

5. Can I live in Duluth and not live in Minnesota?

LESSON TWO: Use other inclusion relations such as:



with similar questions asked.

CLASSIFICATION: LOGIC: ATTRIBUTES

LESSON ONE

Introduction: If you have ASCOBLOCS, use transparency versions of the large pieces - twelve of them. Make a transparency 3 x 4 or matrix (see master forms). Show the children all of the blocks and then place 3 blocks on the matrix. Example:

		R
	Y	
B		

Pick up one of the remaining pieces and show it. Ask the children where it should go on the matrix. Repeat until all pieces have been placed. Discuss why the pieces are placed where they are placed.

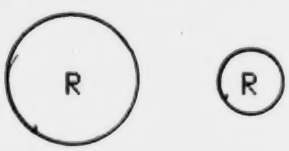
Activity: Pass out worksheets and blocks and have groups of 3 or 4 children work on completing the matrices.

If you have attribute blocks use a 4 x 4 matrix and only the large transparencies of the blocks. Do as above. Example:

			G
		R	
	B		
Y			

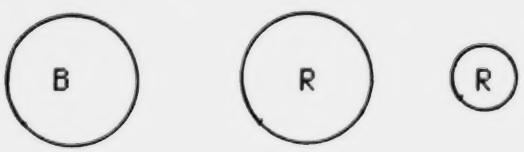
LESSON TWO

Introduction: On the overhead, place an attribute piece. Next to it place a piece that DIFFERS on just ONE attribute. Example:



differs ONLY in SIZE

Ask the children if another piece differs from the Large Red circle in just ONE way. Elicit the idea this could be either SHAPE or COLOR. Place the piece they suggest on the other side.



Activity: Group the children in threes or fours with the worksheet and a set of attribute blocks. Have them continue the train until all blocks are used.

The attached form can be used to print attribute blocks on heavy colored paper or light colored board like railroad board. There are 4 shapes of 2 sizes and as many colors as you choose to use.

ATTRIBUTE BLOCKS (Similarities & Differences): LOGIC 1

LESSON ONE:

Background: The best materials to use at this level to introduce children to the meaning of "AND", "OR" and "NOT" are logic blocks in the form of ASCOBLOCS. The solid polyethylene materials have color, shape, size and thickness. Restrict these to two attributes to introduce the ideas to children. Initial activity should have emphasis on SIMILARITIES AND DIFFERENCES between the objects in the collection.

Introduction: Use overhead transparency materials that are like the ASCOBLOCS. The children should have the large, thick ASCOBLOCS - twelve of them. Place one of these on the overhead. Ask the children to hold up a piece that is the SAME shape as yours. Discuss this. Isolate the pieces of one color, i.e. red. There will be red pieces that are of four different shape - circle, square, rectangle, triangle.

Ask the children to hold up a piece that is the SAME color as yours. Discuss this.

Ask the children to hold up a piece that is a DIFFERENT shape from yours. "How many shapes are there DIFFERENT from yours?" Do the same with color.

Activity:

A. Have the children sort their ASCOBLOCs into two groups.
    1. Could they do this? Why or why not?

B. Have the children sort their blocks into three groups.
    1. What property was used?
    2. How are the blocks in the same group DIFFERENT?
    3. How are the blocks in the same group ALIKE?

C. Have them sort the blocks into four groups and explore the same kinds of questions.

D. Use the balance of the time to have them make composite shapes of the blocks, see which have a dimension in common, which fit together to make another shape, etc.

ASCOBLOCKS (NOT): LOGIC 2

Introduction: Place a loop of string on the overhead. Arrange the large thick simulations in transparency film so all squares are INSIDE the loop and other blocks outside. Ask the children to use TWO words to describe those outside the loop. Reinforce "NOT square" when that comes up.

Activity: Give the children string loops and large thick ASCOBLOCS (or large if they are cardboard facsimiles of the ASCOBLOCS so thickness cannot be represented). Have them place the blocks inside and outside the loop to complete the worksheet.

The pupil book has pages involving similarities and differences and a recording sheet for this lesson.

ESTIMATION

Children need to have experience in ESTIMATING quantities. Before they find accurate quantitative values of any kind, ALWAYS ask them to estimate the outcome first. The count of things in a collection - on a surface, in a container, etc; the measurements of length, area, volume; the heights or depths of things; the duration of time are all results that should first be estimated.

Estimating can be combined with graphing. Some examples:

1. Estimate the number of tablespoons of sand, salt, rice, etc. in a jar. Group the estimates into decodes and graph with cards having the estimates. Repeat with different size jars.

2. Estimate the number of small cups of water, juice, nectar, that will be poured from a container and graph. Repeat with different size containers.

3. Estimate the number of UNIFIX cubes to go around the edges of a table. Graph estimates form into tens and ones and record actual number.

4. Estimate how high a ball will bounce. The estimates can be marked on a long vertical sheet of wrapping paper.

5. Estimate the number of washers needed to balance a piece of fruit, a block of wood, etc. and measure.

6. Estimate how long a train or link a given collection of UNIFIX cubes, or paper clips or plastic links will make.

7. Estimate how many sidewalk "blocks" are on the way from home to school. Graph both of these estimates and the counts that the children bring back.

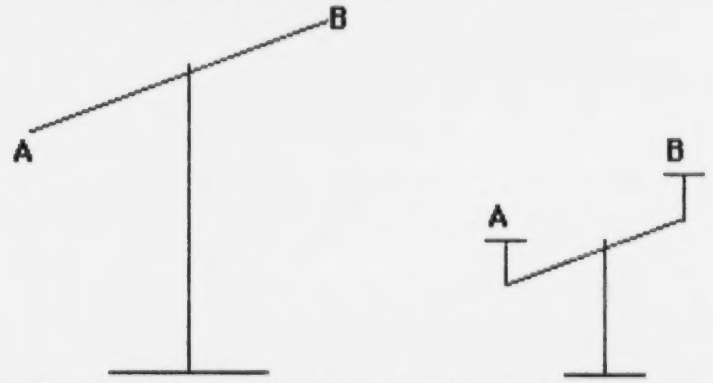
8. Estimate the number of inch cube blocks that would fit into an open top box - then put them into it.

9. Estimate how long it would take to do something - pile up a given number of blocks, link together a given number of UNIFIX cubes, etc. Use a metronome or swings of a pendulum to mark the time.

ORDERING

LESSON ONE

Introduction: Illustrate how to use a balance to decide which of two objects is heavier. Either a hopper balance or a beam balance is to be used. As an illustration show the children 2 blocks from a geoblock set that are unlike, but not too different in weight. Ask the children to estimate which will tip the balance in a direction you specify.



Weigh the blocks and then place them in order with the lighter on the left. Tell the children they are to find a balance and 3 objects at the stations set up. They are to record the order in which this should be placed by weight.

Activity: Set up a balance at each of five stations with a different collection of 3 objects at each. Label the yellow pattern block "A", UNIFIX or MULTILINK link of cubes "B", a ceramic tile "C", a base ten long "D", a washer "E" and a geoblock "F". Arrange these as follows:

STATION	1	2	3	4	5
ITEMS	A,B,C	A,B,D	B,C,D	B,D,E	D,E,F

Pair the children. Each team is to go to each station with the recording form and decide the weight ordering of the objects.

Follow Up: Compare the results. Seek explanations for why they ordered as they did. Is there any way they can use what they found to order four of them? five of them? all six of them?

USE OF SYMBOLIC FORMS

Children cannot use symbols or attach meaning to symbols without

(1) Development of the related concept through extensive experience at the concrete level;

(2) Careful attachment of symbols to the concepts; and

(3) Generation of symbolic forms to show understood concepts

prior to interpretation of symbolic forms. The relative amounts of time to be spent at each stage of this learning of the use of symbols is shown below:

Concept Development at the Concrete	Connecting Symbols to Concepts	Generating Symbolic Forms	Interpreting Symbolic forms in in materials	Responding to Symbolic form with symbols
50%	20%	10%	10%	10%

Technically: numerals and +, - , =, etc. are "signs." They become symbols when the user has attached these to concepts, so that they symbolize for him some real situation.

NUMBER SENTENCES: SOME GAMES

LESSON ONE: Put Together Game

This is similar to the hand games in Mathematics Their Way. It should be combined with oral problems that you give and for which children arrange materials and give number sentences orally and in writing.

MATERIALS NEEDED: Counters (beans, UNIFIX cubes, wooden cubes), workmats, recording forms.

The Game: The number of counters used is at the option of Player 1. The children exchange roles. The children work in pairs. Player 1 takes a chosen number of counters, she puts some into each hand under the table or behind her back. Then she shows the contents of one hand. Player 1 describes these. Then Player 1 shows the contents of the other hand. Player 2 identifies these and orally describes what number of counters that Player 1 started with.

Example: Player 1

- -

Player 2

"five" "plus four" "equals nine"

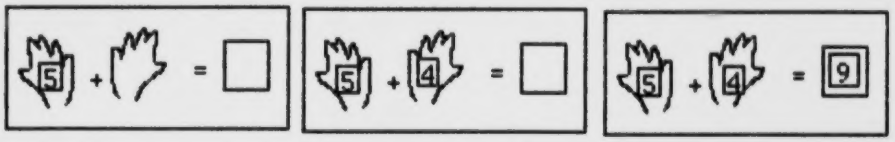
When the mechanics of this game are mastered, introduce the workmats and numeral cards. Player 2 now:

1. orally describes the change; and
2. places numeral cards on the workmat.

Example: Player 1

Player 2

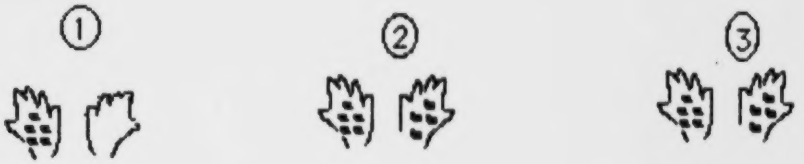
"five" "plus four" "equals nine"



When the children have connected numerals and operation signs to the numbers and changes in this way, introduce the recording form. Now Player 2 will:

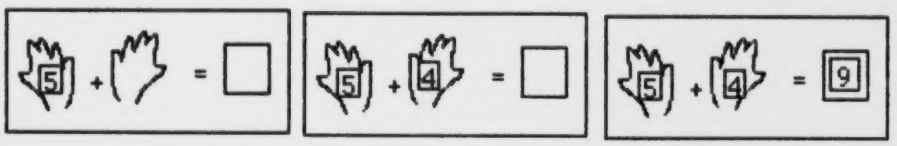
1. orally describe the change; and
2. WRITE numerals onto the recording form.

Example: Player 1



Player 2:

"five" "plus four" "equals nine"

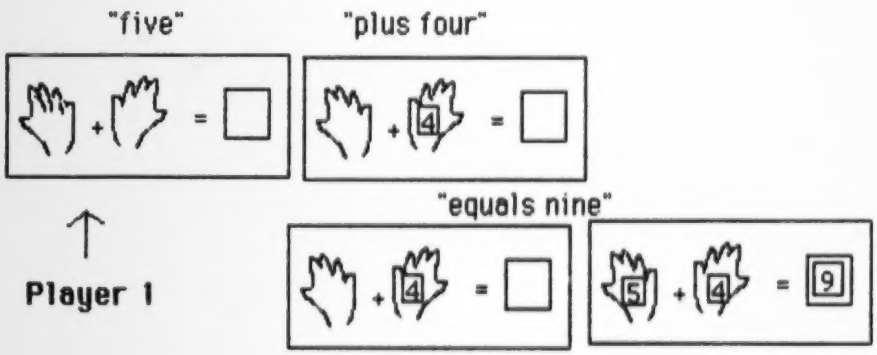


Extensions:

1. Player 1 closes the first hand after Player 2 has orally described the contents and the sequence is repeated in the three stages.

Example: (connecting level) Player 1

Player 2



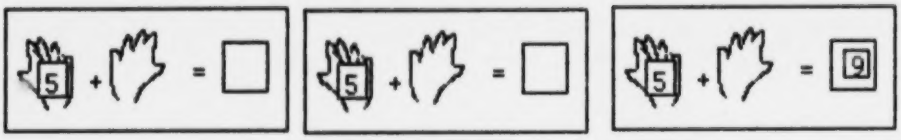
Player 2: Writes numerals on recording sheet

2. Player 2 extends one hand showing the number of counters. Player 2 describes the amount. Player 1 extends the other closed. Then Player 1 opens that hand and Player 2 describes what was in the closed hand.

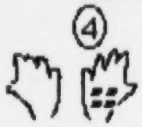
Example: (connecting level) Player 1

Player 2

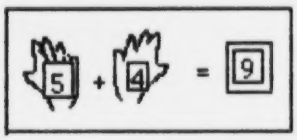
"five" "plus?" "equals nine"



Player 1:



Player 2:

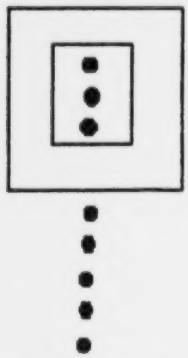


LESSON TWO: The Combine Game
This game is to be coordinated with the oral problem activities done with the entire class. It is designed for pairs of children. It gives experience with both kinds of combine problems. It is adapted from activities in Mathematics Their Way.

MATERIALS NEEDED: Counters (beans, UNIFIX cubes, wooden cubes) "window frames" cut from construction paper or poster board, recording forms.

The Game: The game is to be played with a given number, e.g. 5. Thus there are "Six" games, "Ten" games, etc. The given number of counters are placed in a line on the desk between the two players. (A dot card could also be used). Player 2 closes her eyes. Player 1 places the window frame so as to partition the row of counters into two parts, and says "Peek."

Example:



"five plus three equals eight"

or

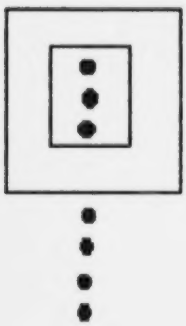
"three plus five equals eight"

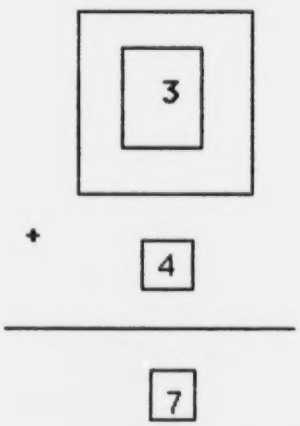
place frame then says,
"Peek"

Player 1 **Player 2**

When the children consistently identify the part-part-whole relationship orally, introduce the second window frame and numeral cards.

Now Player 2 must (1) orally describe the relationship and (2) place numeral cards to show the relationship: Example:





Player 1

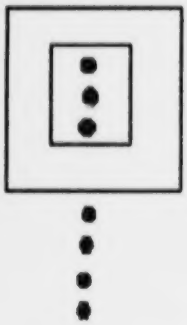
Places window frame over counters

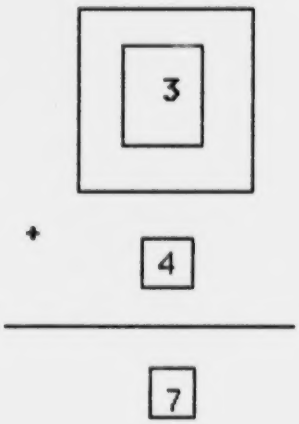
Player 2

"three plus four equals seven" and places numeral cards on workmat

When children see this connection of parts connected by "+" to make the whole, introduce the recording form. Now Player 2 describes the relationship orally and WRITES the numerals on the recording form.

Example:





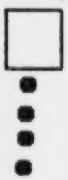
Player 1

Player 2

1. "three plus four equals seven"
2. WRITES numerals on the recording form

Extension: Player 1 uses a solid window and reminds Player 2 of the total number of counters. Player 2 responds as shown.

Example: (concept level)

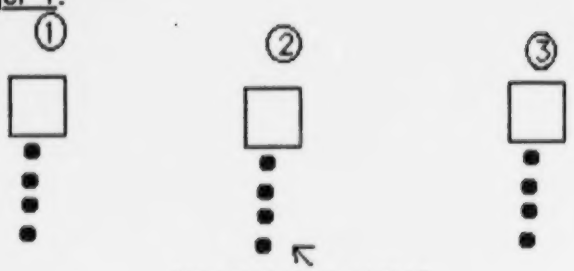


"The total is eight"
Player 1

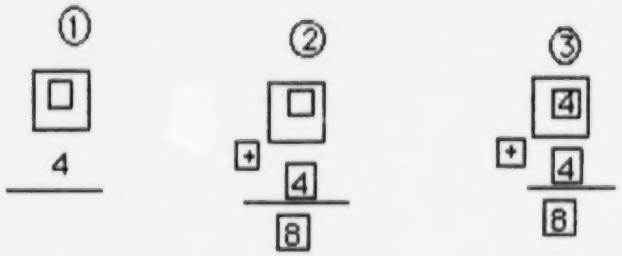
"The missing part is four"
"Four plus four equals eight"

(connecting level)

Player 1:



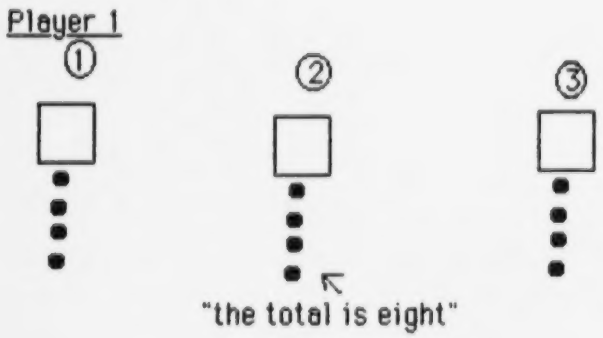
Player 2:

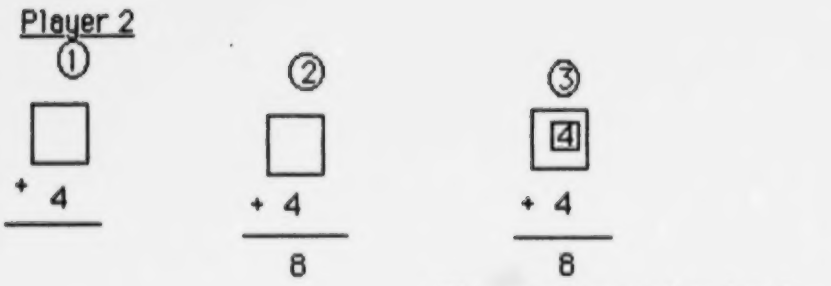


"The missing part equals four"
"Four plus four is eight"

When this is well thought through by the children and cards are placed correctly, introduce the recording form for Player 2.

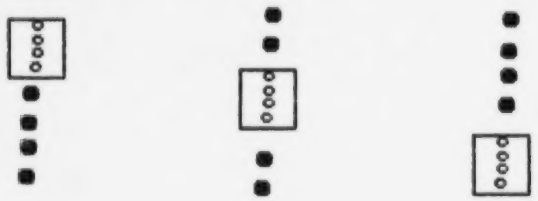
Example:





"The missing part is four"
"Four plus four equals eight"

Player 1 should vary placing the card, ie.



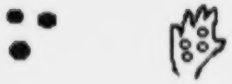
This activity leads to (1) a vertical writing of the addition and (2) both kinds of Combine problems.

LESSON THREE: The Take-Apart Game
This is similar to the hand games that are described in Mathematics Their Way. It is played in pairs by the children and should be combined with using materials to solve orally stated problems that involve separating.

MATERIALS NEEDED: Counters (beans, UNIFIX cubes, wooden cubes), recording forms.

The Game: Each game is played with a given number of counters, e.g. 7, so there are "five" games, "eight" games, etc. The given number of counters is placed on the desk between the two players. Player 2 closes his eyes. While these are closed, Player 1 covers some of the counters and moves them to the side, keeping them covered. He tells Player 2 to open his eyes. Player 2 must describe what happened : "You took 4 from 7 because there are 3 left."

Example:



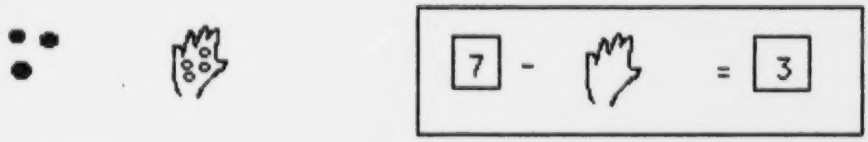
You took 4 from 7 because there are 3 left

Player 1 Player 2

When this goes well, introduce the work mat and numeral cards. At this connecting level, Player 2 will:

1. orally describe the results; and
2. place numeral card to show the amount "taken away."

Example:



Player 1 Player 2 orally describes and places numeral cards as shown and places 4 on the hand. When children are connecting these symbols well, introduce the recording forms and Player 2 writes the numerals on this form.

Player 1

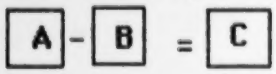
Player 2
1. orally describes
2. writes numerals on recording form

The children should exchange roles so that each pair has equal time as Player 1 and Player 2.

You must judge how much this game must be played by how well children freely write number sentences in response to your orally stated problems. If there is any difficulty with Change 2, 4 and 6 problems, then more of this game activity is needed.

LESSON FIVE: The Difference Game

Introduction: Prepare an overhead transparency of the workmat for The Difference Game. Introduce the game as follows:



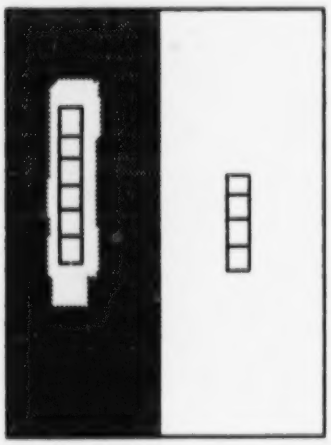
Place a link of ten UNIFIX cubes on each of A and B. Have the children do the same on their workmats. Ask the children if the number in A is the same as the number in B. Turn the projector off; remove three UNIFIX cubes and turn it back on. Ask the children how many cubes are in your hand. Tell them that this is "the difference" between A and B. Ask them how much more is in A than in B. Ask how much less is in B than is in A. Ask the children to write the number in A on the recording form.

MATERIALS NEEDED: 2 colors of UNIFIX cubes per player-pair. A 2 color split board, a lamenated recording form.

The Game: Each game is played with links of a specific number of UNIFIX cubes, e.g. 5. This number can vary from one game to the next so that there are "FIVE" games, "EIGHT" games, etc.

The players sit on opposite sides of a desk with a split board between them. Player 1 puts a 6-link of cubes on the dark half of the mat, which should be to Player 2's left. Behind his back or under the desk, Player 1 breaks off some cubes from a second 6-link and places the remainder of the link on the white side of the board. Player 2 is to mentally compare the two links and state "the difference between (larger) and (smaller) is _____."

Player 1 shows the link in his hand to check the answer. Example at the concept level:



"The difference between 6 and 4 is 2."

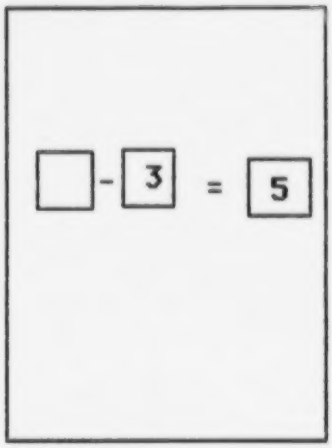
Player 1's placement of the links on the board

Player 2's oral response

The players should be encouraged to place the two links side by side to check the accuracy of the comparison:

Example: (connecting level)

Player 1
"the difference is 5", places the link on the board and arranges numeral cards as shown on the recording form

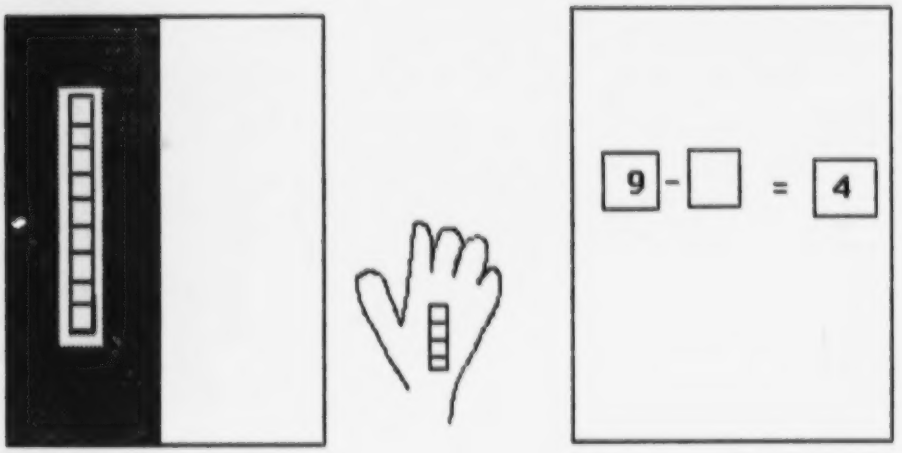


Player 2
"the difference between 8 and 3 is 5", and places 8 in the empty space on the recording form

At the symbolic level, this can also be done by writing numerals on a recording form. Several recording forms would be required.

Player 2 places the link (9, for example) on the dark half of the split board. He breaks off a link from another 9-link and shows it in his hand to Player 2, saying "the difference is (4)." Player 2 arranges numeral cards on the workmat form at the connecting level and writes into a recording form at the symbolic level.

Example:



Player 1
"the difference is 4"

Player 2
"the difference between 9 and 5 is 4", and places numeral card 5 in the empty box

This game should be used as often as necessary depending on the children's responses to orally given problems. If they have difficulty writing correct number sentences for Comparing1 problems, more of this activity is in order.

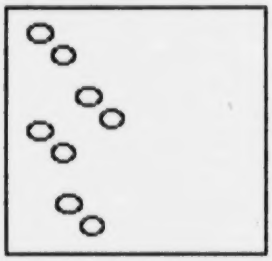
The following black line masters are for workmats to be used for these games and worksheets to be used as a follow-up to the games. The pupil book contains several examples of these worksheets. The master is for you to make any additional exercises as you see necessary for the class or for individuals.

LEVEL TWO

ENCODING NUMBER SENTENCES: FROM MATERIALS

Introduction: As you tell a number story involving addition or subtraction, show the children on the overhead how to use objects to represent things in the problem.

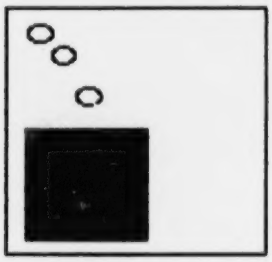
Example: "John had eight game disks for the computer." (Here are John's eight disks):



Write

8

"John gave some of his disks to his sister." (Cover up some of the objects):



Write

8 - ☐ =

"John had only three disks left." How many did he give to his sister?":

Write

8 - ☐ = 3

The children should have laminated poster board or lap chalkboards and objects such as UNIFIX, tiles, etc.

As you read them number "stories", making sure they are exposed to all FOURTEEN kinds that lead to addition and subtraction over the year, have them arrange materials to represent things in the problems and write the

number sentences after each "story", then have them hold up the number sentence written so you can see these.

Don't be in a hurry. It will take time for children to use symbols to represent concrete objects.

Accept EQUIVALENT sentences that have "+" and "-" in them to represent the situation. Have them circle the number that is the answer to the question in all cases.

LEVEL TWO

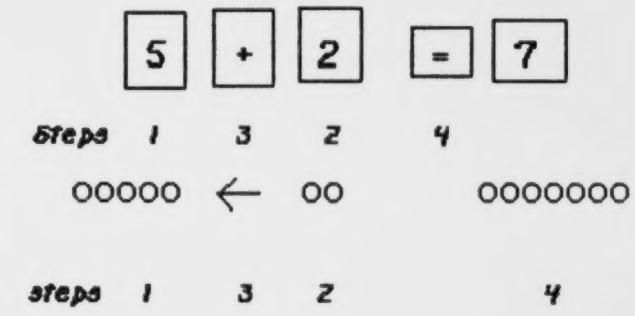
DECODING NUMBER SENTENCES: INTO MATERIALS

Background: After children have learned to write number sentences to represent arrangements of real materials and operations with real materials, they can begin to decode number sentences into arrangements of materials and into actions upon material. Any countable materials can be used for this. Vary these - UNIFIX, beans, etc.

Introduction: Write a numeral on the overhead projector or chalk board. Ask the children to show this many with their materials. Then write a second numeral and ask them to show this. Then write an operation symbol between, "+" for example, and ask the children to do something with the materials to show this. Then write "=" and ask the children to suggest a number to number sentence.

Write an incomplete number sentence and ask the children to do something with the materials to show this and suggest the number to complete the sentence. Pass out the worksheet and have the children use materials to show these and complete the sentences. Walk about and observe what children are doing and that the number sentences are being correctly interpreted into materials.

Examples:



Extensions: When base ten representation is understood, introduce representing sentences having two-digit numerals with base ten blocks.

LEVEL TWO

COMPUTATION FORMS

Background: Children should be adept at changing a horizontal computation form (number sentence) to a vertical form and vice versa.

Lesson One

Introduction: Write an open number sentence on the chalkboard. Ask the children to complete it. Then write the same "fact" in vertical form and ask children to complete it. Point out that these two forms show the same number relationships. Give several such examples, then have the children complete the worksheets. Have counters available for them to use in doing this.

Examples:

Horizontal	Vertical
$2 + 3 = 5$	$\begin{array}{r} 2 \\ +\,3 \\ \hline 5 \end{array}$
$6 = 5 + 1$	$\begin{array}{r} 5 \\ +\,1 \\ \hline 6 \end{array}$
$8 + 5 = 13$	$\begin{array}{r} 8 \\ +\,5 \\ \hline 13 \end{array}$

In using vertical computation forms, children should be encouraged to "look for a ten", i.e. 2 from the 5 along with 8 make ten so 3 from 5 is left and the result if ten + three or 13. Attached are masters for forms to use to develop exercises that will have children go forth and back between horizontal number sentences, and vertical computation forms. The pupil book has several of these that are already prepared.

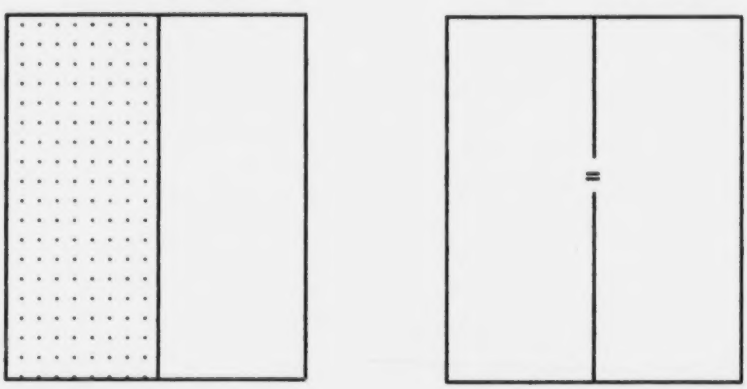
LEVEL TWO

EQUALITY

Background: Children will have had some experience with the three kinds of equality experiences at Level One, but this must be more extensive at Level Two.

Any countable, uniform material - UNIFIX cubes, beans, buttons, bottle caps, wooden cubes - should be used. Split boards should be made. These can be the reverse side of the Place Value boards used early in the year.

SIDE ONE *OBVERSE SIDE*



When children are familiar with the base ten blocks these can be used in equality activity also. Differing numbers of objects should be used - eight one day, fourteen on another, etc.

LESSON ONE: The same number of objects are placed on both sides of the equality board. The LEFT side is kept constant. Children group the RIGHT side into TWO groups in as many different ways as they can and write a number sentence of the form: a = b + c each time.

Things to observe:

Use of symbols - numerals, +,=.
Correctness of number sentences

Question children about meaning of "same as" and "equals." Emphasize that the two sides of number sentence are two ways of writing the same number, i.e., "9" and "5 + 4" are two representations for the same number.

Extensions: Have the groupwed side put into THREE groups with number sentences generated into FOUR or FIVE groups with number sentences generated into EQUAL groups with number sentences generated.

Examples of number sentences:

12 = 7+5 12 = 5+7 12 = 9+3 12 = 10+2

12 = 8+4 12 = 4+8 12 = 3+9 12 = 2+10

12 = 11+1 12 = 2+11 12 = 6+6 12 = 2x6

12 = 3+4+5 12 = 2+2+8 12 = 3+6+3

12 = 2+3+4+3 12 = 4x3 12 3x4 12 6x2

12 = 12x1 12 = 1x12

LESSON TWO This lesson involves keeping the RIGHT constant and grouping the LEFT side in a variety of ways as in Lesson One. Examples of number sentences:

4 + 5 = 9 5 + 4 = 9 6 + 3 = 9 3 + 6 = 9 2 = 7 = 9

7 + 2 = 9 8 + 1 = 9 1 + 8 = 9 9 + 0 = 9 2 + 3 + 4 = 9

1 + 5 + 3 = 9 3 x 3 = 9

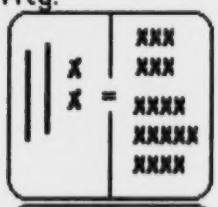
LESSON THREE: This lesson involves grouping BOTH sides, sometimes in the same way such as TWO Groups and sometimes in different ways such as LEFT into two groups and RIGHT into THREE groups or into EQUAL groups. This lesson provides for the greatest flexibility in use and really shows children how many different ways the same number may be expressed. Continually point out the two sides of the number sentences are names for the SAME number. Examples of number sentences:

4 + 2 = 5 + 1 1 + 2 + 3 = 1 + 5 4 + 5 = 3 x 3

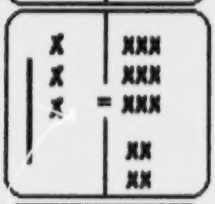
6 + 5 + 1 = 2 + 2 + 3 + 5

LESSON FOUR: This involves using place value represented numbers on one side - LEFT or RIGHT, and counters, grouped or ungrouped on the other. Some

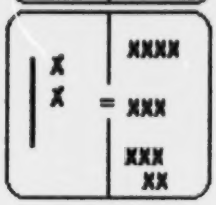
examples of split boards show the variety of experiences possible with this activity:



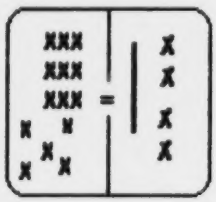
22 - 3 x 3 + 4 + 5 + 4
22 - 9 + 13
20 + 2 = 9 + 13



13 - 9 + 4
13 = 3 x 3 + 2 x 2
10 + 3 = 3 x 3 + 2 x 2



12 - 4 + 3 + 5
10 + 2 = 7 + 5



9 + 5 = 10 + 4
9 + 5 = 14

Attached are masters for recording equality board activities and generating number sentences for equality relations. These are also included in the pupil books.

Following are examples of addition and subtraction problems as used in Level One.. Use larger numbers in these. They exemplify the fourteen situations that give rise to these two operations.

Also included are a smaller number of examples of multiplication and division problems that involve joining, separating and comparing. Create others like these as the need arises, adjusting number size to reflect what students are capable of handling. Also included are some multi-step and more than two number problems as examples.

A good resource for good problem solving situations that emphasize all of the strategies - guess and check, use of tables, making lists, drawing pictures, searching for patterns and determining operations to use from numberless language problems are the Problem Solving books by Charles and Lester that Addision Wesley publishes.

INTRODUCING WORD PROBLEMS

Introduction: Children must know which operations are suggested by language used. Actions such as joining and separating are suggested by certain language, while static situations involving part-whole and comparison are suggested by other language.

Children should be introduced to these uses with problems that have no given numbers in order to concentrate on the meaning of the language.

LESSON ONE: Children should have lap chalkboards and UNIFIX cubes. They are to write the operation symbol, "+", or "-", to show what they think should be done with the given quantities in the problem on the lapboard and hold it after you have read the problem.

Read each problem slowly to the children. Observe the operation symbols displayed and discuss each problem to find out why the children are writing each operation sign.

Problem One: "Charlie found some nuts. Jane gave him some more nuts. How many nuts did Charlie have then?"

This is a simple joining and + shows the operation to be performed on the numbers had same been given. The form is:

A + B = ☐ or ☐ = A + B

LESSON TWO: "Bobbie had some marbles. He gave part of his marbles to Jackie. How many marbles did he have left?"

This is a simple separating and "-" shows the operation linking the two given quantities. The form is:

A - B = ☐ or ☐ = A - B

LESSON THREE: "Billie had some soda pop straws. His sister gave him some more. I know how many Billie had then. How many did his sister give him?"

This is a missing addend situation:

A + ☐ = B

Since it is a joining, some children will write "+". Others may realize what must be done with the given A and B and write "-". Ask the children to explain why they wrote what they did and discuss these differences.

LESSON FOUR: "Charlie Chipmunk gathered some nuts. He gave part to his sister and had some left. I know how much he had left. How many nuts did Charlie give to his sister?"

This is a separating of the type:

A - ☐ = B

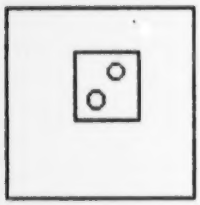
The separating is shown by "-" and "-" is performed on the given quantities so either way the children should show you "-".

LESSON FIVE: "Corrine found some berries on a plant and put them in a pail. Paul gave her his berries to put in the pail. I know how many berries were then in the pail. How many berries did Corinne have to start with?"

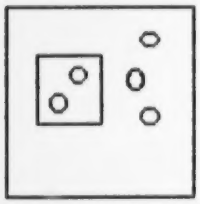
This is of the form:

☐ + A = B

"+" shows the joining and "-" shows what to do with A and B. This will require extensive discussion, with some small numbers for examples. Perhaps the use of chips on the overhead would be needed:



Add three chips:



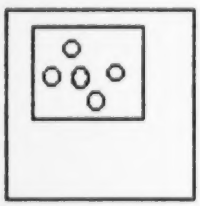
"Now there are five chips. How many are under the card?"

LESSON SIX: "Polly had some stuffed animals. She gave part of these to her sister and counted how many she had left. How many did she have to start with?"

This is a separation of the form:

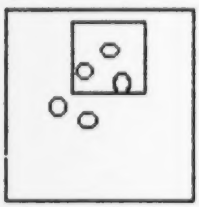
- A = B

The separation is shown by "-". "+" shows what to do with the known quantities, so children again might display either sign. Discuss this. Do an example on the overhead if needed.



Chips under a card

Lift the card slightly and remove a few



Lift the card to show how many are now there and ask the children how many were under the card at the beginning.

LESSON SEVEN: "Susie Squirrel found some red leaves and some brown leaves for her nest. How many leaves did she find altogether?"

This is a simple part-part whole with NO ACTION indicated. The form is:

A + B = ☐

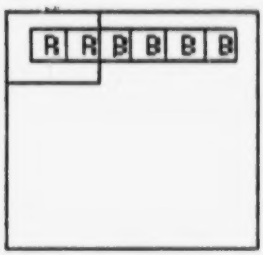
Children should show a + to indicate combining the parts to find the whole. If need be, show a 2 color UNIFIX link and compare this with the language in the problem.

LESSON EIGHT: "Charlie Chipmunk had some red berries and some blue berries. I know how many red berries he had and how many berries he had altogether. How many blue berries did Charlie have?"

This is a part-part whole where a missing part must be found. If children fully understand + separate parts, they will write "+". Some may write "-" to show what to do with A and B in:

A + ☐ = B

Again you may have to make a row of chips on the overhead:



Cover the red with a card before showing the display. "Some of the row of chips are red and some are blue. I have six chips in all. How many of the chips are red?"

$\square + 4 = 6$ shows this, but $6 - 4$ will give the $\square$ value.

LESSON NINE: "While picking berries, John saw some bears. Patty saw fewer bears than John. How many more bears did John see?" This is finding the difference in a comparison and - will show this.

$A - B = \square$

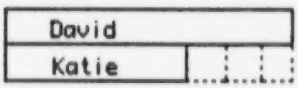
If necessary show a red cube link and a shorter blue cube link to model this.

LESSON TEN: "There were some robins in a tree. In another tree were some bluebirds. There were more bluebirds than robins. There were how many fewer robins?"

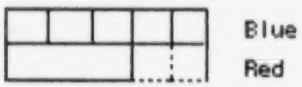
This is another find the difference comparison ($A - B = \square$), but with the opposite comparison language. You will have to really spend time contrasting these two situations.

LESSON ELEVEN: "Katie has some flowers. David has 3 more flowers than Katie. I know how many David has. How many flowers does Katie have?"

This is a comparison where the difference is given and the smaller must be found so - will show this. You may have to model this with UNIFIX links.



"There are 5 blue cubes and 2 more blue cubes than red cubes. How many red cubes are there?"



LESSON TWELVE: "Freddie Fox put some leaves in a pile. Frannie Fox had 4 fewer leaves in her pile. How many leaves are in Frannie Fox's file?"

This again gives the difference (4) and the larger quantity, so - is the operation to find the smaller quantity. Again you may have to give several examples with "more than", "fewer", "less than" language.

LESSON THIRTEEN: "Tony had some boxes. He had 3 more boxes than Tom. I know how many Tom has. How many does Tony have?"

In this comparison the difference and the smaller quantity are given, so + is needed to find the larger quantity. Use UNIFIX links and several additional examples to distinguish this from the two previous cases.

LESSON FOURTEEN: "Susie Squirrel has some sunflower seeds. She has two fewer than Charlie Chipmunk. I know how many Susie has. How many does Charlie have?"

Again, the difference and the smaller are given, so + is used to find the larger. The difference is in how the comparison is stated A compared with B, or B compared with A..

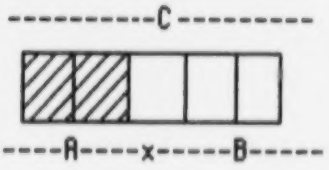
Those problems in Lesson One, Lesson Two and Lesson Seven have proved to be easiest for children to interpret correctly. Those in Lesson Three, Lesson Six, Lesson Nine, and Lesson Fourteen have proved to be the most difficult for children just beginning school and well into third grade, when reversibility of thinking is more likely to be found. Work harder on these more difficult situations for the children.

LEVEL TWO

MORE WORD PROBLEMS:

Background: All arithmetic operations are based on joining, separating, recognition of part-part-whole and comparing. Some joinings involve questions that require subtraction; some separatings involve questions that require addition, etc. There are 14 basic and different addition and subtraction types that result from using two quantities, either equal or unequal.

Part-part-whole



Known	Wanted	Sentence	Operation Needed on Known Quantities
A,B	C	□ = A + B	Addition
A(or B),C	Other Part	C = □ + B	Subtraction

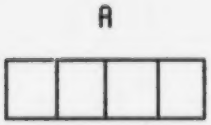
JOINING

Known	Wanted	Sentence	Operation Needed on Known Quantities
A,B	C	A + B = □	Addition
A,C	B	A + □ = C	Subtraction
B,C	A	□ + B = C	Subtraction

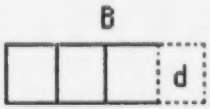
SEPARATING

Known	Wanted	Sentence	Operation Needed on Known Quantities
C,A	B	C - A = □	Subtraction
C,B	A	C - □ = B	Subtraction
A,B	C	□ - A = B	Addition

COMPARING



1) "less than" language used to describe "d".



2) "more than" language used to describe "d".

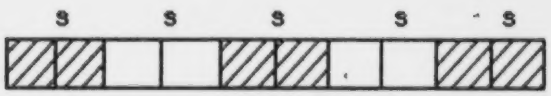
Known	Wanted	Sentence	Operation Needed on Given Quantity
A,B	d	A - B = d	Subtraction
A,d	B	A - □ = d	Subtraction
B,d	A	□ - B = d	Addition

3 for each of (1) and (2), totalling 6 cases.

There are 16 basic and different multiplication and division types. Those that are primarily part-part-whole, joining, separating and comparing activities involving EQUAL groups being joined and separated are described below.

Part.....Part-whole

several
(equal parts)

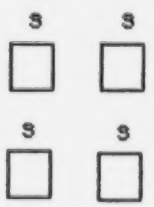


T is the whole of these equal parts

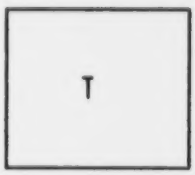
n of these

Known	Wanted	Sentence	Operation Needed on Given Quantities
s,n	T	T = s x n	Multiplication
s,T	n	T = s x □	division
n,T	s	T = □ x n	division

JOINING (several equal sets)



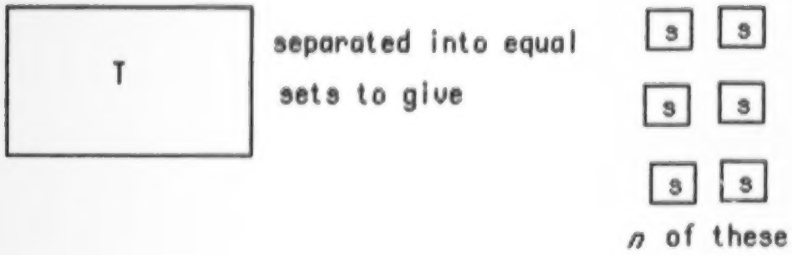
joined to give



n of these

Known	Wanted	Sentence	Operation Needed on Given Quantities
s, *n*	T	T = s+s+s+...s or T = s x n	Multiplication
s,T	*n*	T = s x □	division
n, t	s	T = □ x *n*	division

Separating (into several equal sets)



Known	Wanted	Number Sentence	Operation Needed on Given Quantities
T,s	n	T = s x □	division

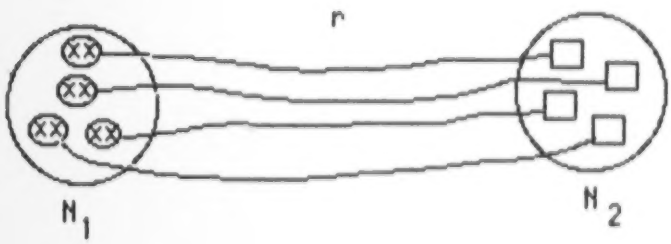
This is the "measurement" interpretation of division - T is measured by known s

T,n	s	T = □ x n	division

This is the "partitive" interpretation of division - T is separated in "n" sets of unknown size

n,s	T	□ = n x	multiplication

Comparing



r is RATIO of x x's to □ 's
or correspondence of x's with □ 's

`1

Known	Wanted	Number Sentence	Operation Needed on Given Quantities
N_1, N_2	r	$\square = \frac{N_1}{N_2}$ or $N_1 : N_2$	division
N_1, r	N_2	$r = \frac{N_1}{\square}$ or $N_1 : \square$	division
N_2, r	N_1	$r = \frac{\square}{N_2}$ or $\square : N_2$	multiplication

Children should have experience with all of these situations that involve part-part-whole, joining, separating and comparing both unequal and equal quantities.

Have them arrange the materials used in response orally given situations. Emphasize what language indicates comparison or looking at existing part-part-whole and language that indicates actions of joining and separating.

LEVEL ONE

WORD PROBLEMS WITH NUMBERS

Introduction: When the fourteen lessons with the situations leading to addition and subtraction have been done often enough so that children can recognize when + and - need to be used in each case, start using numbers in these situations.

PROCEDURE: Children should have UNIFIX cubes to use to represent quantities in the problems.

Stage 1: Read the first problem slowly. After the children have used the UNIFIX cubes to find the answer, call on one child to give the answer orally. Ask the children whether they added the two numbers together or subtracted one from the other. Ask them to give a number sentence for you to write to show what is happening in the problem. Reinforce the meaning of the symbols. Proceed with other problems from the other lessons. Circle the number that answers the question in each case in each number sentence written.

LESSON ONE (1)
2 + 1 = (3)

LESSON TWO (1)
6 - 4 = (2)

LESSON THREE (1)
2 + (6) = 8

LESSON FOUR (1)
4 - (2) = 2

LESSON FIVE (1)
7 = (3) + 4

LESSON SIX
(9) - 2 = 7 OR 7 = (9) - 2

LESSON SEVEN (1)
6 = 3 = (9)

LESSON EIGHT (1)
9 = 3 + (6)

LESSON NINE (1)
④ = 7 - 3 OR 7 - 3 = ④

LESSON TEN (1)
③ = 8 - 5 OR 8 - 5 = ③

LESSON ELEVEN (1)
⑦ - 3 = 4

LESSON TWELVE (1)
5 - ② = 3

LESSON THIRTEEN (1)
7 - 3 = ④

LESSON FOURTEEN (1)
⑥ - 4 = 2 OR

Children should learn to always show the "difference" number in the comparison alone on one side of the number sentence. This provides (1) a focus on difference descriptions "more than", "less than", "fewer than", and (2 a consistent approach to the comparison.

Stage 2: Use the recording form supplied.

Read each problem slowly once after instructing children to concentrate on the question and finding the number that answers it. These should be entered on the recording form. Then read the problem slowly a second time after instructing children to think of the number sentence that shows what is in the language of the problem. This should be written in that space. Thirdly, have children circle the number in the number sentence that answers the question in the problem.

These number stories should be presented to children at least once each week. Below are some examples of number stories that can be used to parallel each of the fourteen situations. Many of these were developed by Nancy Berg and Carol Dyson, two primary teachers in Duluth, MN. You can develop similar stories of these 14 types to use. Mix these up so children get 3 or 4 different situations during each math period. Take time to discuss each. Show the correct number sentence on the overhead, with the correct number circled.

LESSON ONE:

1. "Kris brought two crayons to school. She found another one in her desk. How many pencils did Kris have?"

2. "During the first fall, the Pilgrms built 2 big log cabins. In the spring, they built 5 more log cabins. How many cabins did they build?"

3. "Donnie took 3 tree ornaments from the Christmas box. His mother gave him 4 more to hang on the tree. How many tree ornaments did Donnie have to hang?"

4. "Jack made 4 paper snowflakes. Tom gave him the 5 he had made. How many paper snowflakes did Jack have in all?"

5. "Jodie had 2 red candy hearts. Jane gave her 6 more candy hearts. How many candy hearts did Jodie have then?"

6. "On Mayday there were 7 kites flying in the sky. Then Fred's class sent their 2 kites up. How many kites were in the sky then?"

LESSON TWO:

1. "Charlie Chipmunk found 6 seeds. He gave 4 seeds to his mother to eat. How many did Charlie have to eat?"

2. "Tom has a box of 8 crayons. He gave 2 crayons to Sue to use. How many crayons does Tom have left in his box?"

3. "Indians who visited the Pilgrims had 5 bearskins. The gave 4 of these to the Pilgrims. How many did they have left?"

4. "Joyce found 7 toy soliders in a box. She gave her brother 4 to hang on the Christmas tree and saved the rest. How many toy soldiers did she save?"

5. "Pedro made 8 snowballs. He threw 5 at a can on the fence. How many did he have left?"

6. "Jill bought 7 valentines. She gave 6 to her classmates. How many did she have left?"

7. "Sam's kite needed more bows to fly. She had 9 bows on her kite's tail. She gave 5 of them to Sam. How many bows were left on her kite?"

LESSON THREE:

1. "2 school buses were parked by the school. Some more buses came and parked. Josie counted 8 school buses parked. How many buses parked late?"

2. "The Pilgrims have 3 sacks of corn to plant. The Indians gave them several more sacks of corn. The Pilgrims planted 6 bags. How many sacks of corn did the Indians give them?"

3. "Dan brought home 5 decorations made in school. Ann gave him those she had made in school when he got home. Dan had 8 decorations to put on the tree. How many decorations did Ann give him?"

4. "Tommy has 2 pairs of boots. His brother gave him those he had outgrown. Tommy now has 4 pairs of boots. How many pairs of boots did his brother give him?

5. "Jerry had 3 valentine suckers. Mack gave him some more. Jerry now has 7 valentine suckers. How many valentine suckers did Mack give him?"

6. "Gary's bows on his kite tail numbered 4. He added the bows Sue gave him. Now there are 9 bows on Gary's kite. How many bows did Sue give him?"

7. "Fannie's mother put 5 candles on the birthday cake. She saw she needed more so she put those on the cake. The right number of candles is 8. How many more candles did she put on the cake?"

LESSON FOUR:

1. "Sophie Squirrel gathered 4 acorns. She hid some under a log. She put the other 2 acorns in the nest. How many acorns were under the log?"

2. "The teacher had 6 erasers. She gave 1 eraser to each child in row 3 to use. She has 2 left. How many children are in row 3.

3. "Dan had 8 pieces of tinsel. He gave some to Ann. He has 2 pieces left. How many pieces did he give to Ann?"

4. "Bennie made 5 snowmen. His dog knocked some snowmen down. Two snowmen are still standing. How many snowmen were knocked down?"

5. "Sally has 8 valentines. She sent some to her friends. Now she has 2 valentines. How many valentines were sent to her friends?"

6. "Francie had 7 pieces of birthday cake. She put some in the cake saver. She gave the remaining 3 pieces to her friends. How many pieces were put in the cake saver?"

7. "Tom picked 9 dandelions on the way to school. He dropped some on the way. He gave 6 dandelions to his teacher. How many did Tom drop?"

LESSON FIVE:

1. "Chuck picked some hazel nuts. His brother gave him 4 more hazel nuts. Chuck had 7 hazelnuts to bring home. How many hazel nuts did Chuck pick?"

2. "Paul brought his marble sack to school. His friend Barnie gave him 3 more marbles. Paul now has 8 marbles. How many marbles were in his sack when he came to school?"

3. "John brought some bread loaves to the Indian village. They already had 4 loaves. The Indians shared 7 loaves of bread. How many did John bring?"

4. "Sally had some dried cranberries on a string. Terri put 3 more cranberries on it. There were 9 cranberries when they put it on the tree. How many cranberries did Sally have at the start?"

5. "Mike had some knit caps. His grandmother gave him 2 more. Now he has 5 knit caps to wear. How many did he have to begin with?"

6. "Kim had some valentines. She made 3 more in school and had 8 valentines to give to friends. How many did she have at the beginning?"

7. "Some kites were flying in the school yard. Tom sent 2 more into the air. Then there were 9 kites flying. How many kites were there at the beginning?"

8. "For her birthday Jane received some bows. She put these with

the 4 bows in her drawer. She has 8 bows now. How many bows did she get on her birthday?"

LESSON SIX:

1. "Susie Squirrel gathered some acorns. She dropped 2 on the way to her nest. She placed 7 in the nest. How many did she gather?"

2. "There was a stack of books on the class table. Mary took 3 of them. That left 2 for Fred to choose from. How many books were on the table at the start?"

3. "The Pilgrims put some wild turkeys in a pen. They took 3 to give to their Indian friends. Then there were 6 turkeys in the pen. How many turkeys did the Pilgrims put in the pen?"

4. "When John went to buy tree ornaments he saw a box with several in it. He bought 2 and counted 6 left in the box. How many were in the box to begin with?"

5. "Several snowflakes landed on Susie's mitten. She watched 5 melt and 3 were left. How many snowflakes landed on her mitten?"

6. "Some valentine candies were in a dish on the table. Steve took 2. When Jane came by she counted 5 candies in the dish. How many candies were in the dish to begin with?"

7. "Some kites were flying. After 3 were pulled down, 6 were still flying. How many were flying to begin with?"

8. "Some presents were in a pile on the table. When his mother gave Tom 2 of these, 5 were left in the pile. How many were in the pile at the start?"

LESSON SEVEN:

1. "Chuckie Chipmunk had 6 red berries and 3 green berries. How many berries did Chuckie have?"

2. "Jeanne has 3 reading books and 2 coloring books. How many books does Jeanne have?"

3. "When the Pilgrims and Indians had their first dinner together,

there were 4 pumpkin pies and 2 apple pies. How many pies did they have?"

4. "In making a wreath for the tree, Janet used 3 red loops and 5 green loops. How many loops did she use?"

5. "Kari has 2 red scarves, 3 white scarves, and 1 blue scarf. How many scarves does Kari have?"

6. "Tom had 2 heart valentines and 4 lace valentines. How many valentines did Tom have?"

7. "Gary found 2 fielder's gloves and 1 first baseman's glove. How many baseball gloves did Gary find?"

8. "Willie received 6 presents wrapped in green paper and 2 wrapped in white paper. How many presents did Willie receive?"

LESSON EIGHT:

1. "Bruno Bear found a bush with 9 berries. 3 were red and the rest were green. How many berries were green?"

2. "Hanging in the cloakroom are 8 jackets. 2 are red and the rest blue. How many jackets are blue?"

3. "On the Thanksgiving table were 5 bowls of vegetables. 3 had squash and the rest had corn. How many bowls had corn?"

4. "Ann had 8 Christmas tree ornaments. 6 of them had sparkles and the others had no sparkles. How many of the Christmas tree balls had no sparkles?"

5. "Tim has 6 pairs of mittens. 2 pairs are wet and the rest are dry. How many pairs of mittens are still dry?"

6. "Of the 7 jelly beans in a dish, 5 were red and the rest were black. How many jelly beans were black?"

7. "9 birds were sitting on the fence. 3 were blue and the rest brown. How many brown birds were on the fence?"

8. "All of Joyce's 7 presents were wrapped in colored foil. 2 were

in silver foil and the rest in green foil. How many presents were in green foil?"

LESSON NINE:

1. "Charlie Chipmunk saw 7 bears. Susie Squirrel saw only 3 bears. Charlie Chipmunk saw how many more bears than Susie?"

2. "Sue has 2 papers to bring home. Tom has 7 papers to bring home. Tom finished how many more papers than Sue?"

3. "For Thanksgiving, the Pilgrims had 6 turkeys and 3 fish. They had how many more turkeys than fish?"

4. "The Christmas tree light set had 8 blue lights and 4 red lights. How many more blue lights were there?"

5. "Jim made 3 snowballs and Fred made 6 snowballs. Fred made how many more snowballs than Jim?"

6. "Kelly received 6 valentines and Jolene received 4 valentines. Kelly received how many more valentines than Jolene?"

7. "When Jack and Sally went to pick flowers, Jack found 9 daisies and Sally found 4 buttercups. Jack found how many more flowers than Sally?"

8. "Jean and her twin sister Joan received birthday presents from their friends. Jean got 8 presents and Joan got 6. Jean got how many more presents than Joan?"

LESSON TEN:

1. "5 blackbirds were in one tree. 8 crows were in a second tree. How many fewer blackbirds were there"?

2. Bill brought 3 toy cars to school. Jim brought 6 toy cars to school. How many fewer did Bill bring?

3. All of the children went berry picking. The Pilgrim children picked 3 baskets of berries. The Indian children picked 5 baskets of berries. The Pilgrim children picked how many fewer baskets of berries?

4. The Christmas tree had 8 red lights and 6 blue lights. There

were how many fewer blue lights?

5. In their snow fort, Pat stored 3 snowballs. Betty had 6 snowballs stored. Pat had how many fewer snowballs in the fort?

6. When the teacher distributed the Valentine mail, Sam received 4 and Jean received 7. Sam received how many fewer valentines?

7. Sue had 9 fresh daisies and Sam had 6. Sam had how many fewer fresh daisies?

8. Tom had 4 candles on his birthday cake and his brother had 9 candles on his. Tom had how many fewer candles on his cake?

LESSON ELEVEN:

1. Charlie has 3 nuts in his pail. Susie has 4 more nuts than Charlie. Susie has how many nuts in her pail?

2. When the class went to the gym, the girls had 4 rubber balls. The boys had 2 more rubber balls. How many rubber balls did the boys have?

3. At the first Thanksgiving the children played games. The girls played 5 games. The boys played 3 more games than the girls played. The boys played how many games?

4. The Christmas tree had 3 presents with bows underneath. The presents without bows were 3 more in number than those with bows. How many presents were without bows?

5. Tom's team had 3 practice pucks. Sue's team had 4 more than Tom's team. How many hockey pucks did Sue's team have?

6. Joyce found 5 dandelions. Kristine found 3 dandelions more than Joyce. How many dandelions did Kristine find?

7. 4 school kites had red ribbon tails. The kits with blue ribbon tails were 3 more than this. How many kites had blue ribbon tails?

8. On her birthday, Tanya received 5 gifts. Janet received 2 more than this on her birthday. How many presents did Janet

receive?

LESSON TWELVE:

1. Charlie had 5 acorns in his pail. Susan had 3 fewer acorns in hers. How many acorns did Susan have?

2. Jim brought his set of 7 toy soldiers to class. Tom had 4 fewer soldiers in his set. How many were in Tom's set?

3. For the first Thanksgiving, the Indian children picked 6 baskets of berries. The Pilgrim children picked 2 fewer baskets. How many baskets did the Pilgrim children pick?

4. A Christmas tree had 9 red lights. There were 3 fewer blue lights. How many blue lights were on the tree?

5. Patty's snowman had 8 buttons. Jean's had 3 fewer buttons. How many buttons were on Jean's snowman?

6. Sam received 7 valentines. Sue received 2 fewer valentines. Sue received how many valentines?

7. Tammy found 5 dandelions. John found 1 dandelion less than Tammy. How many dandelions did John find?

8. Tom and Gail had birthdays two days apart. Tom's cake had 7 candles. Gail's cake has 2 candles less than Tom's. How many candles were on Gail's cake?

LESSON THIRTEEN:

1. Susie Squirrel has 7 animals in her family. She has 3 more animals than Charlie has in his family. How many animals are in Charlie's family?

2. Jean has 6 peanuts. She has 3 peanuts more than Pam. Pam has how many peanuts?

3. Indian children looped 7 poles. This was 3 poles more than the Pilgrim children looped. How many poles did the Pilgrim children loop?

4. Tom's chair had 9 presents on it. This was 3 presents more than Jane's chair. Jane had how many presents?

5. The Lost and Found Box in Room 10 had 8 pairs of mittens. This was 3 more pairs than Room 12 had. How many pairs of mittens were in Room 12's Lost and Found Box?

6. 8 valentines had flowers on them. This was 2 more than the valentines with hearts. How many valentines had hearts on them?

7. 8 kites flew on the sunny side of the school. This is 3 more than flew on the shady side. How many kites flew on the shady side?

8. Tom's 7 pencils for his birthday were 2 more than Jane's ribbons for her birthday. How many ribbons did Jane have?

LESSON FOURTEEN:

1. Joyce has 4 sunflower seeds. She has 2 fewer than Charlie. How many seeds does Charlie have?

2. Joe choose 3 library books. He chose 3 fewer than Deanna. How many library books did Deanna choose?

3. The Pilgrims had 5 ponies. They had 2 ponies fewer than the Indians. How many ponies did the Indians have?

4. Linda selected 5 of the ornaments. She selected 3 fewer than Tom. How many ornaments did Tom select?

5. Tony made 2 small snowmen. He made 3 fewer snowmen than Kathy. Kathy made how many snowmen?

6. Jake received 5 valentines. He received 3 fewer than Joannie. Joannie received how many valentines?

7. Bob's bag had 6 marbles. He had 3 fewer marbles than Gene. Gene had how many marbles?

8. Andy's 5 birthday presents were 2 fewer than his sister Gail's. Gail had how many birthday presents?

In creating and using these kinds of "story problems," use as many real objects as found in the problems as you can. Some suggestions are:

Beginning of school: Pencils, crayons, paper sheets, scissors, books notebooks and similar objects associated with this period of time.

Fall: Acorns and other nuts, leaves, seeds, and other things found on the ground at this time of year.

Winter: Snowmen, skates, skies, snowballs, mittens, tassel caps, jackets, overshoes and other things associated with this time of the year.

Spring: Kites, flowers, birds and other things related to this time.

Birthdays: Candles, presents, gifts, friends, etc.

Thanksgiving: Turkey cutouts, pumpkin cutouts, corn, etc.

Christmas: Unbreakable tree ornaments, gifts, tree lights, wreaths, bows.

Valentine's Day: Hearts, valentines, valentine candies

Easter: Jelly beans, eggs, paper rabbits, baskets

LEVEL ONE

MORE WORD PROBLEMS: Adding and Subtracting

1. Beth had 3 red balls. Jeff gave her 3 more red balls. How many red balls did she have then?

2. Ann had 8 flowers. She gave 5 of them to her mom. How many flowers did she have left?

3. Pam had 2 books. Sam gave her some more books. Then Pam had 5 books. How many books did Sam give her?

4. Jan had 10 sticks. She gave some to Sid and then had 4 sticks left. How many sticks did she give to Sid?

5. Bill had some cars. He gave 4 cars to Jack. Now he has 1 car left. How many cars did he have to start with?

6. Pete had some cans of pop. Jeff gave him 3 more cans of pop. Now he has 6 cans of pop. How many cans of pop did he have to start with?

7. Val has 5 green bows and 4 red bows. How many bows does Val have together?

8. Jake has 7 turtles. 5 are big and the rest are little. How many turtles are little?

9. Tom has 7 trucks and Sam has 6 trucks. How many more trucks does Tom have then Sam?

10. Becky has 7 rings. Jackie has 3 rings. How many fewer rings does Jackie have then Becky?

11. Ted has 7 kittens. Bill has 2 more than Ted. How many kittens does Bill have?

12. Todd has 9 pairs of socks. Ted has 2 fewer pairs of socks than Todd. How many pairs of socks does Ted have?

13. Sara has 4 marbles. She has 3 marbles more than Jean. How many marbles does Jean have?

14. Bob has 4 pencils. He has 4 fewer pencils than Jan. How many pencils does Jan have?

15. Rick has 1 rabbit. He has 4 fewer rabbits than Ron.
How many rabbits does Ron have?

16. Pam has 7 dollars. She has 2 dollars more than Jim.
How many dollars does Jim have?

17. Ron has 6 turtles. Sid has 4 fewer turtles than Ron.
How many turtles does Sid have?

18. Val had 1 dollhouse. Ann has 1 more dollhouse than Val.
How many dollhouses does Ann have?

19. Debbie has 5 birds. June has 9 birds.
How many fewer birds does Debbie have than June?

20. Joan has 4 pencils and Mike has 3 pencils.
How many more pencils does Joan have than Mike?

21. Betty has 9 rabbits. 6 are brown and the rest are white.
How many rabbits are white?

22. Mike has 6 trucks and 4 trains.
How many trucks and trains does Mike have altogether?

23. Gail had some beads. Jane gave her 6 more beads. Now she has 9 beads. How many beads did she have to start with?

24. Sam had some pigs. He gave 2 pigs to Frank. Now he has 7 pigs left. How many pigs did he have to start with?

25. Liz had 3 dogs. She gave some to Bob and then had 1 dog left. How many dogs did she give to Bob?

26. Kate had 1 sucker. Jill have her some more suckers.
Than Kate had 8 suckers. How many suckers did Jill give her?

27. Tom had 4 fish. He gave 4 of them to Bob.
How many fish did he have left?

28. Bob had 1 stick of gum. Val gave him 1 more stick of gum. How many sticks of gum did he have then?

29. Sam had 5 bugs. Tom have him 1 more bug. How many bugs did he have then?

30. Beth had 7 candy bars. She gave 4 of them to Ted. How many candy bars did she have left?

31. Jean had 3 balloons. Sue gave her some more balloons. Then she had 6 balloons. How many balloons did Sue give her?

32. Ted had 7 toy jets. He gave some to Ben and then had 5 toy jets left. How many toy jets did he give to Ben?

33. Tom had some hot dogs. He gave 2 hots dogs to Jan. Now he has 2 hots dogs left. How many hot dogs did he have to start with?

34. Pat had some marbles. Jill gave her 4 more marbles. Now she has 10 marbles. How many marbles did she have to start with?

35. Ron has 3 dogs and 2 cats. How many dogs and cats does Ron have altogether?

36. Jenny has 10 coins. 3 are dimes and the rest are nickels. How many coins are nickels?

37. Tina has 3 dolls and June has 9 dolls. How many more dolls does June have then Tina?

38. Mom has 8 oranges. Dad has 6 oranges. How many fewer oranges does Dad have then Mom?

39. Jan has 4 shells. Sam has 3 shells more than Jan. How many shells does Sam have?

40. Max has 6 fish. Jill has 5 fewer fish than Max. How many fish does Jill have?

41. Matt has 10 bugs. He has 1 bug more than Ben. How many bugs does Ben have?

42. Jack has 4 tops. He has 6 fewer tops than Tim.
How many tops does Tim have?

43. Liz has 3 oranges. She has 3 fewer oranges than Val.
How many oranges does Val have?

44. Joan has 8 necklaces. She has 2 necklaces more than Barb.
How many necklaces does Barb have?

45. Sue has 7 bananas. Rick has 1 fewer banana than Sue.
How many bananas does Rick have?

46. Jack has 6 ice cream cones. Liz has 4 ice cream cones more than Jack. How many ice cream cones does Liz have?

47. Rick has 2 dogs. Sue has 4 dogs.
How many fewer dogs does Rick have than Sue?

48. Mary has 10 blouses and Jill has 4 blouses.
How many more blouses does Mary have than Jill?

49. Ken has 9 apples. 2 apples are red and the rest are green.
How many apples are green?

50. Sara has 1 big duck and 6 little ducks.
How many ducks does Sara have altogether?

51. Todd had some dimes. Sue gave him 7 more dimes. Now he has 8 dimes. How many dimes did he have to start with?

52. Ann had some hats. She gave 1 hat to Ted. Now she has 9 hats left. How many hats did she have to start with?

53. Ann had 5 dresses. She gave some to Beth and then had 3 dresses left. How many dresses did she give to Beth?

54. Todd had 5 shells. Joan gave him some more shells. Then Todd had 9 shells. How many shells did Joan give him?

55. Rick had 10 rocks. He gave 3 of them to Pat.
How many rocks did he have left?

56. Jim had 4 apples. Jan gave him 3 more apples.
How many apples did he have then?

57. Pam had 5 shells. Liz gave her 4 more shells.
How many shells did she have then?

58. Joy had 5 dolls. She gave 2 of them to Val.
How many dolls did she have left?

59. Max had 7 ducks. Jim gave him some more ducks.
Then Max had 8 ducks. How many ducks did Jim give him?

60. Tim had 9 ships. He gave some to Val and then had 7 ships left. How many ships did he give to Val?

61. Pam had some rings. She gave 5 rings to Liz. Now has has 4 rings left. How many rings did she have to start with?

62. Jack had some frogs. Tom gave him 1 more frog. Now he has 4 frogs. How many frogs did he have to start with?

63. Joy has 4 blue crayons and 4 yellow crayons.
How many crayons does she have altogether?

64. Matt has 6 balls. 4 are blue and the rest are green.
How many balls are green?

65. Ben has 2 packs of bubble gum and Beth has 5 packs of bubble gum. How many more packs of bubble gum does Beth have than Ben?

66. Jenny has 3 balloons. Jack has 10 balloons.
How many fewer balloons does Jenny have than Jack?

67. Tim has 5 trucks. Tom has 2 trucks more than Tim.
How many trucks does Tom have?

68. Kate has 10 stickers. Jenny has 3 fewer stickers than Kate.
How many stickers does Jenny have?

69. Fred has 9 Transformers. He has 3 Transformers more than Chad. How many Transformers does Chad have?

70. Ann has 4 lollipops. She has 5 fewer lollipops than Pat. How many lollipops does Pat have?

71. Beth has 5 apples. She has 2 fewer apples than Sam. How many apples does Sam have?

72. Al has 5 kites. He has 2 kites more than June. How many kites does June have?

73. Pat has 8 pairs of shoes. Joy has 4 fewer pairs of shoes than Pat. How many pairs of shoes does Joy have?

74. Pam has 3 skirts. Beth has 5 skirts more than Pam. How many skirts does Beth have?

75. Jeff has 8 airplanes. Todd has 9 airplanes. How many fewer airplanes does Jeff have than Todd?

76. Pete has 8 shirts and Tim has 5 shirts. How many more shirts does Pete have than Tim?

77. Barb has 8 ice cream cones. 6 are chocolate and the rest are strawberry. How many ice cream cones are strawberry?

78. Jeff has 2 bats and 8 balls. How many bats and balls does Jeff have altogether?

79. Kate had some pet rabbits. Joy gave her 5 more pet rabbits. Now she has 7 pet rabbits. How many pet rabbits did she have to start with?

80. Jill had some dolls. She gave 3 dolls to Sue. Now she has 5 dolls left. How many dolls did she have to start with?

81. Ron had 8 apples. He gave some to Joy and then had 6 apples left. How many apples did he give to Joy?

82. Chad had 4 guns. Tim gave him some more guns. Then Chad had 7 guns. How many guns did Tim give him?

83. Ben had 6 trucks. He gave 5 of them to Jack. How many trucks did he have left?

84. Max had 2 blue cars. Tim have him 3 more blue cars.
How many blue cars did he have then?

LEVEL TWO

PROBLEM SOLVING: WITHOUT NUMBERS

Introduction: Read a "story" problem that has no numbers in it. Ask the children what number operation would be needed on the given things to answer the question.

Example:
"John had some chestnuts. He gave some of his chestnuts to Fred. How many chestnuts did John have left?"

Talk about what is given and what needs to be found. What operation is done on the given numbers to find the answer.

Do this periodically throughout the year, sampling ALL fourteen of the + and - situations.

Activity: Give children cards with "+" and "-" on them. Read "story" problems to them one at a time. The children are to hold up a card to show which operation to do or the given numbers.

WORD PROBLEMS: More Than Two Numbers

Introduction: Introduce this extension to the children with a story. They should have UNIFIX cubes or other material to represent the materials. Ask them to use the materials and answer the question in the problem.

"Tom has three balloons; Bill has four balloons and John has two balloons. How many balloons do they have together?"

"Jean had three cookies and Patricia had two cookies. Jean gave one cookie to Theresa. How many cookies did Jean and Patricia have together?"

"Tom had eight marbles. He had two more than John. John had two more than Bill. How many marbles did Bill have?"

"Mr. Pirelli makes tires for bicycles and tricycles. He made enough for 4 bicycles and 3 tricycles. How many tires did he make?"

Devise as many problems involving adding and subtracting that show joining, separating, comparing and part-part-whole as you can.

Combine two of these situations into these problems as much as possible.

GENERATING WORD PROBLEMS

LESSON ONE

Introduction: Present the class with two numbers, 5 and 8, for example. Ask the children to write a story using these two numbers and either addition or subtraction. Give an example: "John has five apples. Corinne gives him eight more. How many does he have now?" Point out that a question must be asked in each story written. When each has written one, ask a few of the children to read their problems. Discuss these. Ask the children which number sentences would be written to show what operation is used to answer the question. Discuss these responses.

Present a second pair of numbers and ask the children to write a story about this pair. Remind the children that one of the numbers can be the result of adding or subtracting. Repeat the previously described discussions. Watch for (1) the variety of situations used, (2) correct use of signs in the associated number sentences, (3) correct use of "left over," "all together," "left," "together," "more than," "less than," and other expressions that are related to the kinds of problems.

LESSON TWO

Give the children three related numbers, e.g. 9, 6, 3. Ask them to write these down and circle one of them. Have them write a story problem that uses the two UNCIRCLED numbers, so that the CIRCLED number will be the answer to the question in the problem.

GENERATING NUMBER SENTENCES

Background: Read ALL of the lessons relating to Word Problems and Number Operations in Level One. Repeat these with larger numbers than recommended for Level One. Keep in mind that the sequence for developing understanding of Number Sentences is:

ORAL DESCRIPTIONS - ARRANGEMENTS OF MATERIALS -
NUMERAL CARD "SENTENCES" - WRITTEN SENTENCES

This same sequence should be used when the numbers are expressed in Place Value form. The materials to arrange then are base ten blocks.

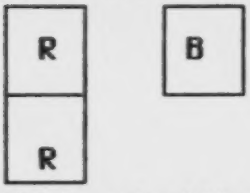
Children will have some difficulty generating number sentences for certain kinds of problems until they have had much experience with all separating, joining and comparing situations. Reread the section of word problems until you thoroughly understand how these give rise to the arithmetic operations that number sentences represent.

CORRESPONDENCES

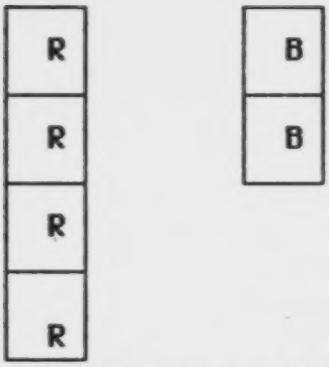
Background: One of the major areas of difficulty for students in intermediate grades and junior high school is in applying ratio and using proportions. An early start in this area at the concrete level would be helpful in preventing some of this.

LESSON ONE

Introduction: On the overhead, use transparent colored squares of two colors. Make a set of TWO of one color and a set of ONE of the other color:



Children should have two colors of UNIFIX cubes. Have them make a TWO link of one color and a ONE link of the other as you have done. Write 2:1 to show this comparison. Then make a second arrangement like the first and join them:

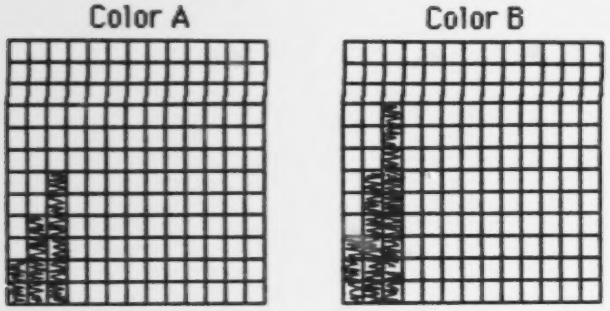


Write 4:2. Point out that the 2:1 groups are still recoverable and that there are twice as many RED as BLUE. Give the children recording forms and have the children complete that.

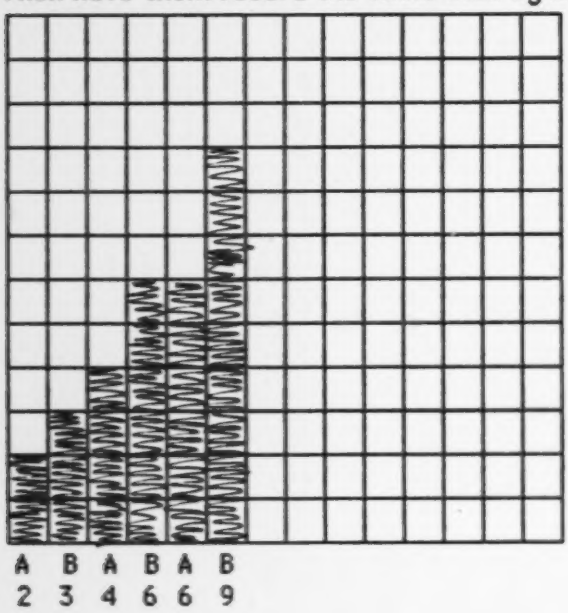
Links of each color	(Red)	(Blue)
1	2	1
2	4	2

LESSON TWO Have the children use UNIFIX cubes to complete recording forms for 3:2; 3:4; 2:3; 1:2 ratios. Repeat this activity once a month or so. Besides giving a concrete example of ratio correspondences, it helps children learn multiplication facts.

LESSON THREE Have children use 2 colors of UNIFIX cubes to build links as in the previous lessons. Provide them with graph paper and have them color in squares to show the links as successively made. The example is for 2:3.



Have them draw lines as shown and compare how the lines are "climbing." Then have them record the links side by side on graph paper:

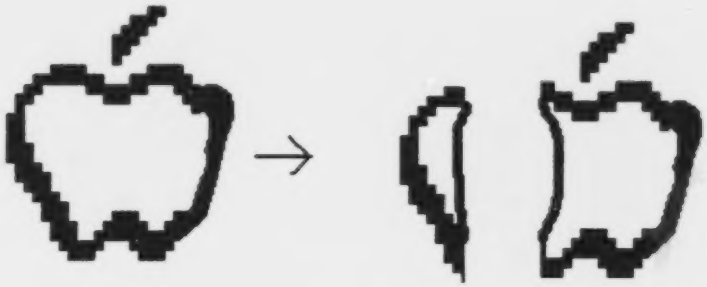


Lines should be drawn connecting the A color corners and also a line connecting the B color corners. Compare the "rise" in the lines. Following is a recording form. Several of these for different ratios are provided in the pupil's book.

LEVEL TWO

FRACTIONS 1

Background: Children have a primitive idea of fractions coming to school. "Half" generally means one of two parts, without regard for the parts being IDENTICAL -



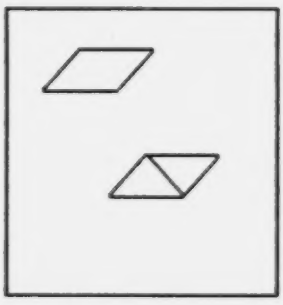
- probably cut in "half" for most children at age 6 or 7. One might recognize the disparity but say, "Your half is bigger than mine!"

Introduction to the idea of fraction is representing some count of several EQUAL: PARTS is through work with concrete materials with such relationships built into them. A fully developed concept of fraction requires eventual use of area, length and volume models. At this level Pattern Blocks are used. Children will have used them previously for several purposes, but at this level, formal introduction to fractions with them is one of the two uses for them.

LESSON ONE:

Group the children in fours with each group having at least 12 greens, 3 blues, 4 reds and 2 yellow Pattern Blocks. Using overhead projector versions of these, show the children what constructions you want done with them.

Example:



"I have made a shape just like the blue piece from two greens. You do the same."

Then:

"A green piece is what part of a blue piece?"
"A blue piece is how many times as big as a green piece."
"Are all of the green pieces equal in size."
"The green piece is what fraction of the blue?"

Have the children build each of the remaining pieces from the same-color smaller pieces in all possible ways such as red from greens, yellow from greens, yellow from reds, yellow from blues. Ask questions about what fraction the red is of yellow, blue of yellow, etc.

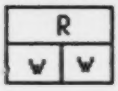
You should continue to question the children about these relationships until they see that:

G = 1/2 B G = 1/3R G = 1/6 Y
B - 1/3 Y R = 1/2 Y B = 2/3 R

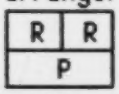
This is necessary since future work will assign Value ONE to the Yellow piece and fractions to the smaller related pieces.

LESSON TWO

Introduction: On the overhead projector, show a red rod. Ask the children to find a rod that is half of red rod. When they respond, arrange 2W below 1R:



Have them find a rod that is half of the purple rod. When they respond, arrange:

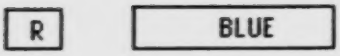


on the overhead.

Have them find half of the dark green rod, then half of the brown rod, then half of the orange rod. Ask which rods have no "half" rods in the set.

Symbolize these relationships as W = 1/2R, R = 1/2P, G = 1/2D, etc. Do the example from the worksheet on the overhead and be sure that they record and color on the worksheet correctly.

Example:



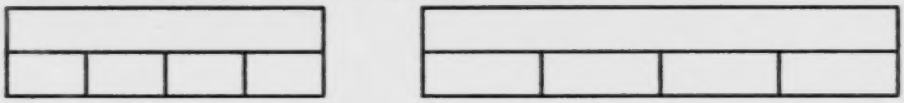
THIRD: none

LESSON THREE

Have the children work with "one third" using the worksheet. Watch to see that they are (1) coloring in on the worksheets and tracing the correct rods for the thirds and halves.

LESSON FOUR

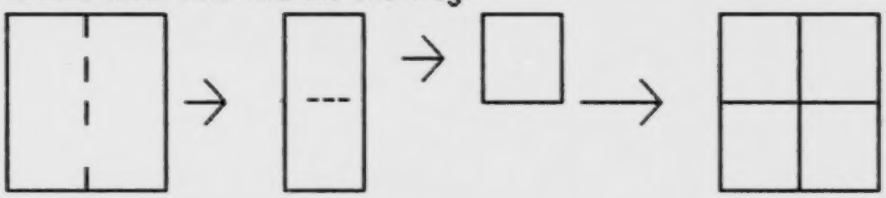
Introduction: Place the purple rod with four white rods and the brown rod with four red rods on the overhead, as shown:



Talk about "one fourth" as being one of four EQUAL parts. The W is one fourth of the P and the R is one fourth of the W rod.

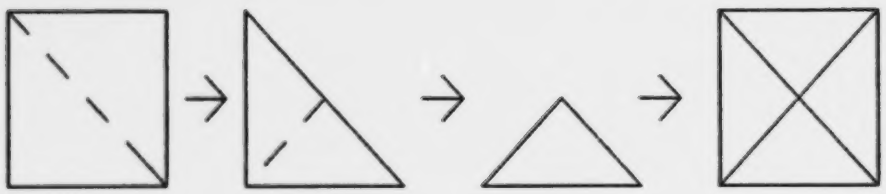
Ask the children for other examples of one fourth, where there are four EQUAL parts.

Activity: Give each child several white squares of paper. Demonstrate how to fold them into fourths one way:



Have them fold three pieces this way and color in ONE fourth, TWO fourths, and THREE fourths.

Demonstrate how to fold it into fourths another way. Talk your way through it.



"Now it's in half"

"Now we have half of half"

Again, have them color in ONE fourth, TWO fourths and THREE fourths. Point out how two fourths = one half.

LESSON FIVE

Activity: Have the children outline a 3 x 3 square on the geoboard. Have them divide it into 2 equal parts with one rubber band and record this on the dot paper. Monitor to see that these are correct. Suggest, "Do it another way." Have them hold up their geoboards so that you can check. If most of them are OK, they should draw these, too. Work with those who are having problems while the others divide the square in as many ways as they can.

Extension: Repeat with a 2 x 3 rectangle, then a 4 x 4 square and a 4 x 3 rectangle. Challenge them to try other shapes, such as triangles or non-rectangular 4-sided shapes.

LESSON SIX:
Use UNIFIX cubes or Cuisenaire Rods to introduce children to 2/3 and 3/4.

Unifix Cubes: Have them make UNIFIX links of a given size of one color. Have them make a second link of a given color that is some fractional part of it. Emphasize the idea of "a certain number of EQUAL parts" as a fraction.

Example:
Make a blue link of 12 cubes.
Make a red link that is 2/3 of it.
Make a green link that is 3/4 of it.

Cuisenaire Rods: Have studentds find shorter rods that are fractional parts of selected rods.

Example:
Find a rod that is 2/3 of a dark green rod.

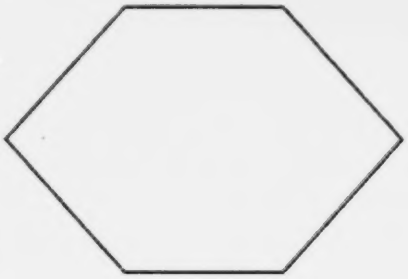
LEVEL TWO

FRACTIONS 2: USING FRACTION WORDS

LESSON ONE

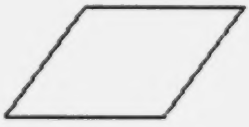
Introduction: Put the yellow hexagon on the overhead projector. Tell the children this is ONE. Make the hexagon from 2 red trapezoids. This, then is ONE-HALF. Make the hexagon from 3 blue "diamonds." Ask the children what fraction a blue piece will be (ONE-THIRD). Make the hexagon from the green triangles. This shape is ONE SIXTH of the yellow ONE.

Write the following alongside each piece.



ONE **1**

ONE HALF **1/2**



ONE THIRD **1/3**

ONE SIXTH **1/6**

Ask the children several comparison questions:

"Is 1/2 more than or less than 1/3?"

"How do you know?"
"Is 1/3 more than or less than 1/6?"
"What else do you know about 1/3 and 1/6?"

Activity: Pass pattern blocks out to the children - 2 of each color. Have the children combine 2 different pieces together in different ways, asking for the resulting fraction (compared to the ONE) each time. Have the children find the difference between (HOW MUCH bigger the larger one is) the red and blue, the blue and green.

Remind the children you join and compare fractions just like you join, separate and compare whole numbers.

LEVEL TWO

PLACE VALUE: UNIFIX CUBE INTRODUCTION TO BASE TEN

Background: Children using this program will have had experience with counting onto a place value mat until a predetermined grouping size < ten was reached, grouped these and placed them in the left column. They also are counting off a place value mat, necessitating decomposing of groups to recover counters. These same games (see Level One Lesson Plans) should be used to introduce base ten. Whereas they previously called groups by names other than the base size, i.e. Bozo for three or Quark for four, TEN is to be given its own name. It is a good idea to make a UNIFIX TEN, and suspend it in front of the room with the label - TEN.

LESSON ONE

Introduction: Children are to each have a split mat and a "bank" of 25 or so UNFIX cubes:

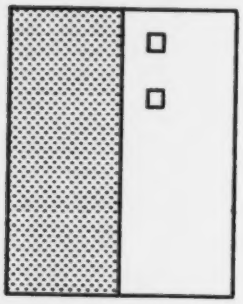
Select a signal to indicate the number to be put on the counting (right) side of the mat - hand clap, rap of a ruler on a desk, etc. Signal to have the students count two, three, four at a time ont the right side. Remind them the TEN is the magic number and when they have TEN UNIFIX they must link them and (1) put the TEN on the left side. At each stage they are to record on the Recording Form.

Example:

Place Value Mat

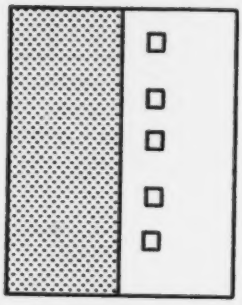
Recording Form

clap, clap



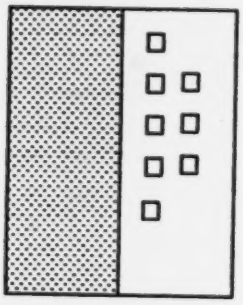
TEN	ONE
0	2

clap, clap, clap



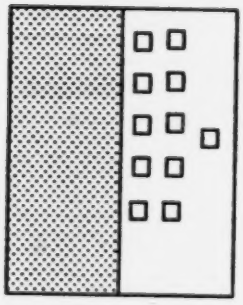
TEN	ONE
0	2
0	5

clap, clap, clap

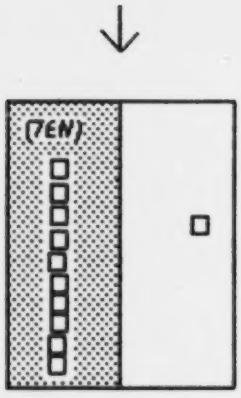


TEN	ONE
0	2
0	5
0	8

clap, clap, clap



TEN	ONE
0	2
0	5
0	8
0	10

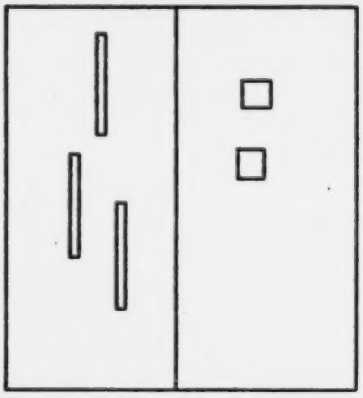


When children are accustomed to this let them play "personal computer" by counting on and recording using a die or spinner as the signal for how much to add to the right column. This can be varied by using TWO spinners or TWO dice, one to indicate tens to add, the other to indicate ones to add. Repeat until concept of "grouping by tens" is well established.

LESSON TWO: Start with a predetermined larger number, i.e. forty-three and use signals to count off the board. This will require students to take apart a UNIFIX TEN to get ONES needed to be able to remove the required number. Be sure they fully decompose the TEN and not remove just enough to have the required ONES.

Follow the same procedures and variations, using the Recording Forms provided.

LESSON THREE: Give the children numbers to show you in tens and one on the place value mat. "Show me thirty-two":



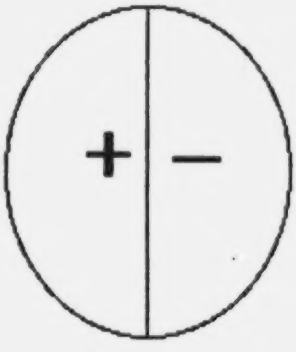
etc.

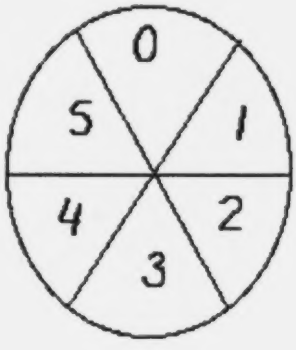
LESSON FOUR: Have pairs of children measure lengths in the room using UNIFIX cubes to make a long train that matches the length. Have them break the UNIFIX cubes into TENS and ONES, place these on a place value mat and record the result on a form:

TENS	ONES	TOTAL

Extensions: Have pairs of children estimate the number of objects in jars or bags - peanuts, marbles, macaroni shells, etc. Then have them count these by TENS in paper cups, place the cups and left over ones onto the place value board and record the results. Compare these with the estimates.

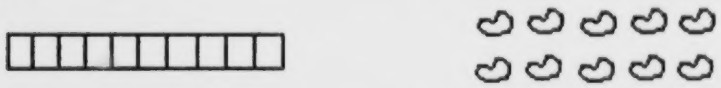
Pair children with UNIFIX cubes (and later - base ten blocks), a +- spinner, a place value mat each and a numeral spinner. They take turns spinning the +/- to see whether to add on cubes or to take cubes off, the numeral spinner to see how many. The object is to get a given number of TENS, i.e., two, three, etc.





LESSON FIVE: This lesson is to provide a transition to trading with base ten blocks.

Introduction: Of the TEN Unifix recording form and the TEN Beans recording form



make a link in two parts of Ten Unifix and color this to show that on the transparency. Then roll ten beans and color that transparency. Write the corresponding number sentences in each case in two ways: 10 = ___ + ___ and ___ + ___ = 10

Activity: Each child should have 12 of each of two colors of Unifix and ten beans that are colored on one side. Have them complete one Beans sheet and one Unifix sheet, writing both kinds of number sentences each time. Remind them to look for tens when they see things like 6 + 7. 4 from the 7 is used with 6 to make ten and 3 is left over, so the result is a ten + three.

It might be helpful to make a Unifix Ten and hang it up in the front of the room with a sign: THIS IS A TEN.

Following are Masters to be used in connection with numeration using UNIFIX cubes. The pupil book has some exercises already prepared using these.

PLACE VALUE: Stations

Set up stations like number stations for students. Use the "My Computer" and other recording forms and worksheets.

Materials to use include:

1. Unifix cubes and Unifix ten frame place value mats with reocrding forms.

2. Base ten blocks and base ten place value mats with recording forms.

3. Beans and cups to use with split boards and recording forms.

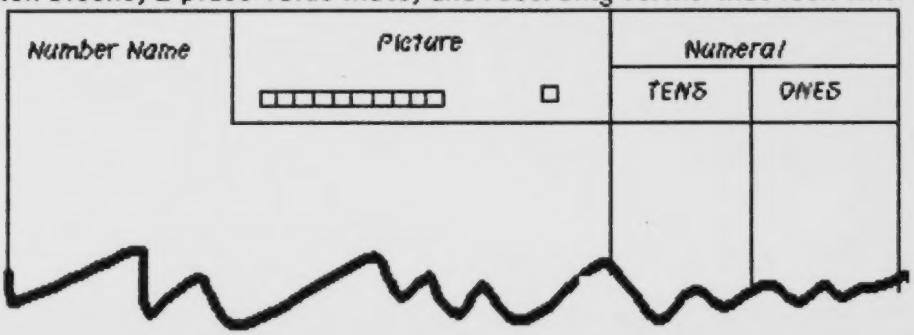
4. Bean stick "tens" to be traded for ten beans accumulated and the appropriate recording forms.

LEVEL TWO

NUMERATION

LESSON ONE

Background: Many children will already have associated counting words with base ten numerals through their experience with page numbers, price tags, etc. This lesson is to clearly associate numerals in base ten with the counting words. Words are to be translated in base ten block representations and then into numerals through pictures. Materials are base ten blocks, 2 place value mats, and recording forms that look like:



Introduction: Read aloud some number between ten and thirty. Ask the children to write that in the NAME column. Have them arrange base ten blocks to represent that number. "Draw a picture in the PICTURE column of your base ten blocks." "Write numerals in the tens and ones columns to show how many of each you used and pictured."

Check each step to see the recording forms are used correctly. Do several different numbers to give children experience.

Student Activity: When children can all correctly use the recording form; pass out the worksheets. They are to use materials to represent the given counting words, draw pictures of their mataerials (a | to represent a TEN block and a * or x to represent a ONE block) and write the numerals. Watch how children make the transition from ORAL counting words to PRINTED counting words.

Extensions: Use this lesson with gradually increasing numbers - twenties and thirties - forties and fifties, etc. Use the recording form as a master to make additional worksheets. Use worksheets that vary the stimulus, i.e.

1. give pictures of base ten blocks - the student writes names and numerals

2. give numerals - student makes pictures of base ten blocks

and writes names.

Use worksheets where each problem has a different stimulus - word, picture, numeral.

LESSON TWO

Have children build counting charts that go from 0 to 99 using the base ten blocks and the Recording Forms.

LESSON THREE

Have pairs of children measure lengths in the classroom using Base Ten TENS and ONES. These should be placed on a place value mat and recorded as done with UNIFIX cube measurement.

Following are masters for recording numeration work with base ten blocks and for making worksheets to use. There are several of these already prepared in the pupils' book.

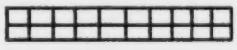
LEVEL TWO

PLACE VALUE-INTRODUCING NUMERATION WITH BASE TEN BLOCKS

Background: Trading ones for tens is different from building tens from ones. The reverse processes of trading tens for ones and decomposing tens into ones are even more different. Trading the tens for hundreds must also be carefully introduced when base ten blocks are used. This will come later.

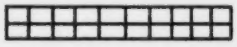
LESSON ONE

Have children build lengths to match the base ten "long" from the "units."

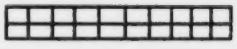


"long"
"units"

Extend this to two longs:



long
units



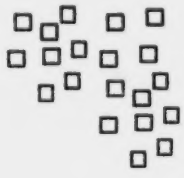
long
units

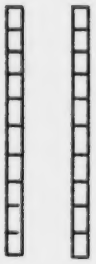
Have them group the "ones" together and the tens together and count:

Point out these are the same -

that TWO TENS show as much

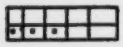
as TWENTY ONES





LESSON TWO

Use cardboard "ten frames" or bean sticks and have the children write the numerals for different combinations of these.

A  - ONE from A is used to make a

B FIVE in B ∴ TEN + TWO or 12

These numerals should be written on recording forms: MAKE LOTS OF THESE!

B

A

Beansticks

Beansticks are made by gluing beans to tongue depressors or popsickle sticks.

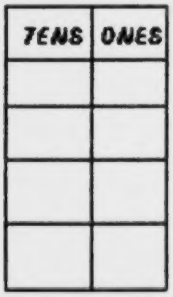
TENS	ONES

LESSON THREE

Do the Counting On activities and Counting Back activities using base ten blocks instead of UNIFIX cubes.

LESSON FOUR

Group the children into fours. Each group should have a collection of base ten ones, tens. Prepare cards with 2 place numerals on them. Give one card deck to each group. The deck is placed face down. One child selects top card and keeps the card hidden while building the number from base ten blocks. Each other child in the group is to record that numeral on a form provided, i.e.:



These could be compared with others in the group and with the "builder's" card. That card is then placed at the bottom of the deck. A second child picks the top card and the process is repeated.

Extension: Use later with 3 digit numerals and hundreds pieces added and a non-columnar recording form.

LESSON FIVE

Use the **My Computer Record 1** and base ten blocks. The child is to complete the record by adding ONE blocks, one at a time to the counting side of a place value mat and enter the number into the record form. Watch for "trades" when tens are reached.

Extension: Use the **My Computer Record 2** AFTER hundreds have been introduced to the children. This lesson can be incorporated into a "station".

LESSON SIX

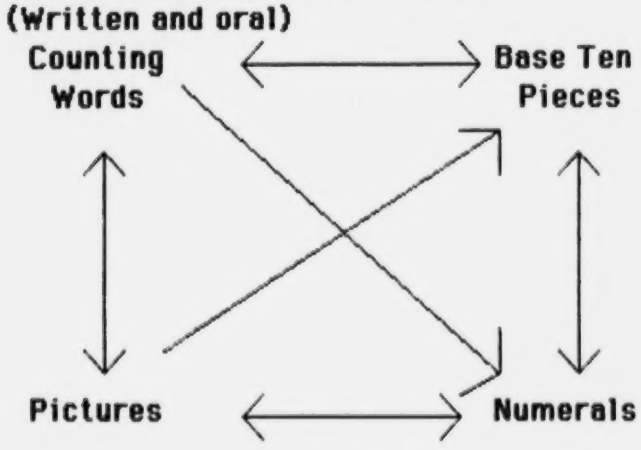
Introduction: Children should have base ten blocks, two spinners or 2 dice. The spinners or dice are to be spun or rolled. One shows how many tens to use - the other how many ones to use in building numbers out of base ten blocks. Demonstrate this with overhead spinners, base ten blocks and the 2 column recording forms until children know what they are to do.

Activity: Have the children complete the "My Computer" worksheet by using one spinner to determine the TENS in the number and the other to indicate the ONES in the number.

Following are Masters for recording forms and worksheets for numeration with base ten blocks that extend into the hundreds.

NUMERATION IN BASE TEN: HUNDREDS WITH BASE TEN BLOCKS

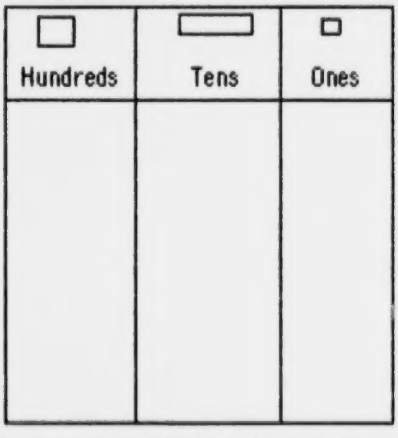
Background: HUNDREDS are not to be introduced until children have mastered TENS into ONES and ONES into TENS. Emphasis should be on the relating of (1) counting words; (2) base ten block arrangements; (3) pictures of base ten blocks; and (4) numerals.



The recording forms and worksheets provided will provide much of this relating, but not all.

LESSON ONE

Introduction: Place a 3 column workmat on the overhead projector:



Place a handful of ONES in that column. Make these into TENs. Trade for base ten TENS and place these in that column. Remove the material.

Place a handful (more than ten) of TENS in that column. Make these into squares or HUNDREDS. Trade these for a HUNDRED and put the HUNDRED in that column. Point out this process is also reversible and HUNDREDS can be traded for TENS.

Activity: Give the children base ten blocks and the workmat called "My Computer Counts to 999." The children are to add ONE UNIT at a time to the ONES column and trade up as needed. At each step the numeral showing what is on the workmat is to be written on the 3 column adding machine paper recording forms, i.e.:

0	0	0
0	0	1
0	0	2
		⋮
0	1	0
		⋮
0	9	9
1	0	0
	⋮	
9	9	9

LESSON TWO

Group the children into fours. Each group should have a collection of ones, tens and a few hundreds in base ten blocks. Prepare cards with 2 and 3 place numerals on them - one/card. Give each group ten such cards. They are to put these in a deck with numerals face down. The first child shows the top card and builds that number with base ten blocks, i.e. :

43 ⟷ **IIII xxx**

Each child is given a turn. The group is to check on other members and correct each other's errors. When the deck is exhausted, exchange decks between groups and continue.

LESSON THREE

Introduction: Children should have base ten blocks including some hundreds, tens and ones and 3 column place value mats.

Write a numeral on the blackboard or on the overhead projector.

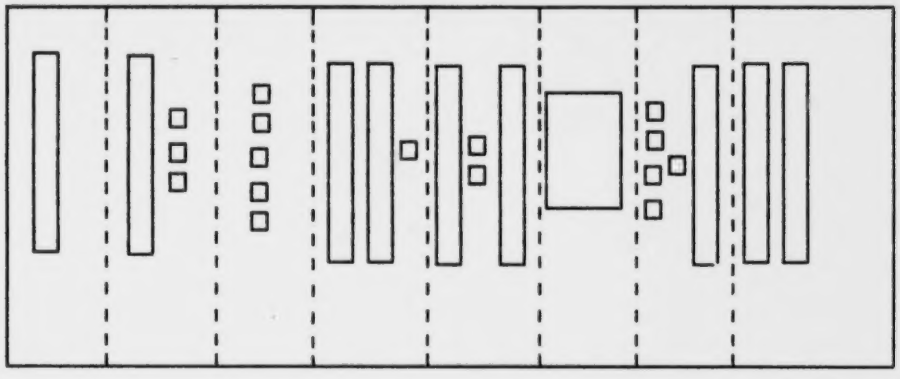
1. Have the children read it aloud;
2. Have the build the number with base ten blocks
3. Have them record the number on the recording form by drawing a picture of the materials and writing the numeral.

Repeat with a couple more examples that include 3 digits.

Activity: Children should use base ten blocks with the worksheets and recording forms. Encourage the use of , 1, and in drawing pictures of the base ten blocks.

LESSON FOUR

Introduction: Make a "mystery box" of base ten blocks on the overhead projector. Cover this with a long piece of cardboard that can be slid back over the base ten blocks once the overhead projector is turned on:



Move the cover to Position 1. Have the children orally describe the number, then write it on the recording form provided. Move successively to Positions 3 through 8, each time having the total shown read, then written on the recording form.

Do a second problem by starting with 7 as the first position and sliding the cover from right to left.

LESSON FIVE

Repeat the procedures above with different arrangements of base ten blocks, omitting the oral descriptions.

LESSON SIX

Students who need additional work - put the base ten blocks on a board and slide another board as a cover away from these with these children one on one.

LESSON SEVEN

Start with a large number on the place value mats, e.g. 88. Do the counting on activity to get to forcing a regrouping into hundreds.

LESSON EIGHT

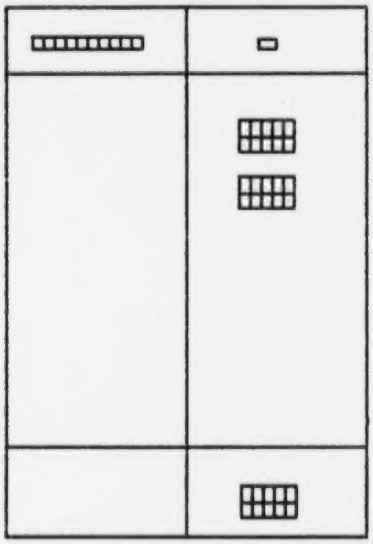
Start with a 3 digit number on the place value mats, e.g. 112. Do the counting back activity to force decomposing a hundred into tens.

COMPUTATION: BASE TEN

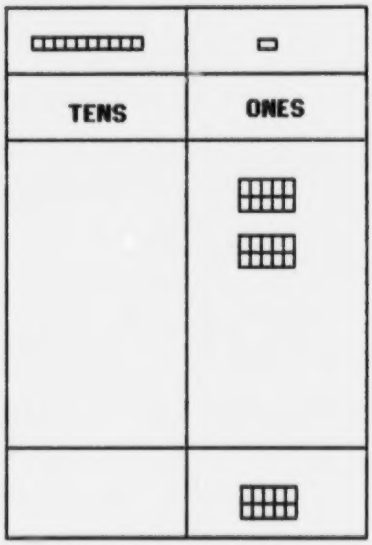
In developing an understanding of the addition and subtraction algorithms in base ten, special work mats and recording forms should be used along with the base ten blocks.

The sequence of place value mats is:

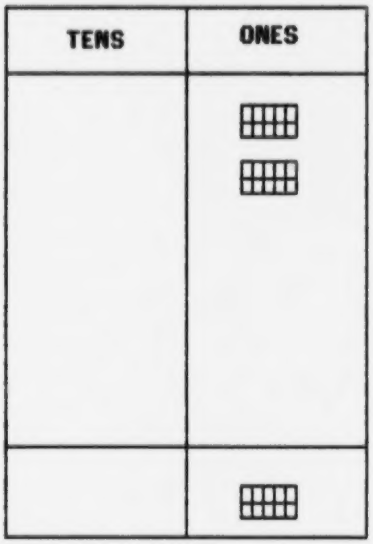
1



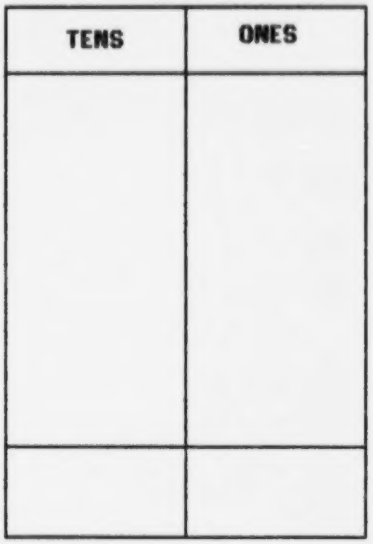
2



3



4



5

HUNDREDS	TENS	ONES

6

HUNDREDS	TENS	ONES

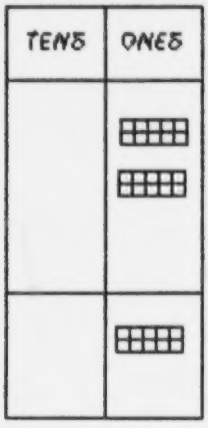
These can easily be made of a heavy white cardboard or poster board, using base ten stamps to make the figures at the column heads. You can do double duty by having an equality board on the reverse side of one and by putting two different versions on the two sides of the same board, e.g. 1 and 2, 3 and 4, 5 and 6. You have been given models for all of these.

In using recording forms and worksheets with base ten blocks, use much the same sequence of column headings. You have been given several samples of these that can be used as black line masters.

LEVEL TWO

COMPUTATION FORMS: PLACE VALUE

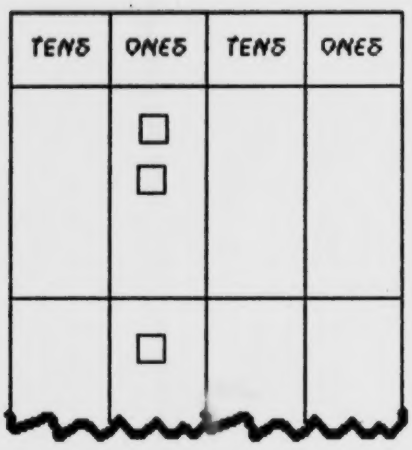
Background: Children must be introduced to computation forms in base ten representation using materials that emphasize the column arrangement of numerals that show group size. Prior to doing this, children must have had experience with lessons to develop place value and to emphasize "making tens," preferrably as two fives by using ten frames. This is a model for a place value mat and for worksheets used in this:



LESSON ONE

Introduction: Use an overhead transparency version of the place value mat, and of base ten blocks. The children should have base ten blocks, place value mats and recording forms.

1. Ask children to place five ONES on the counting side of the place value mat. Have them write 5 on the recording form:



Do the same on the overhead version of the recording form.

2. Have them write "4" below the "5". Remind them you have not yet indicated what should be done with these.

3. Check to see they have done this.

4. Point out the horizontal line. Tell them this is to separate the 5 & 4 from the result, much as the = sign does.

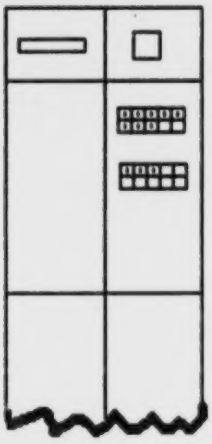
5. Ask them to describe the sign to show these should be joined. Do not have them write it. Then ask which sign would be used if we took 4 from 5. What sign would be used to show how much bigger 5 is than 4? What sign to show how much smaller 4 is than 5?

6. Actually model all of these and write the related sign on the overhead recording form each time. Do this with several number pairs.

LESSON TWO

Introduction: Use the overhead transparency version of the place value mat and base ten blocks. Children should follow by using base ten blocks on their place value mats.

Put an addition problem involving ones on the mat:



Ask the children which sign you should put there.

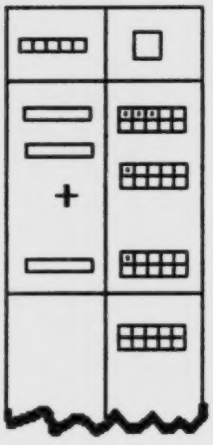
If "+" is chosen, ask the children what to do with the ONES. If they tell you to join them, point out this would "overflow" the ten frame. What should be done then? Exchange ten ONES for a TEN and place it below the line. Then move the remaining ONE(S) below the line. Point out that they should always look to make a TEN out of the ONES and place a TEN in that column.

If "-" is chosen, ask the children what to do with the ONES. If "take away" is suggested, ask what to "take away" from what. Move any ONE(S) that remain below the line. If comparison is suggested, make a 1-1 correspondence between ONES in the ten frames and move the surplus ONES below the line.

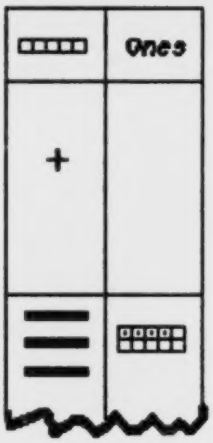
Activity: Pass out worksheets and have the children do these using place value mats and base ten blocks.

LESSON THREE

Introduction: Use the overhead place value mat. Place the following on the overhead:



Ask the children what the "+" tells you to do with the materials. Physically join them by moving the Base Ten TENS together and the ONES into one of the ten frames. Ask the children where the result of the joining should go. Move these below the line, keeping the ones in the ten frame:



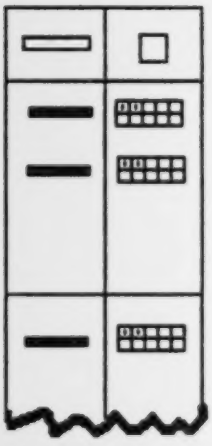
Do one more like this.

Activity: Have the children use place value mats and base ten blocks to complete the worksheets involving addition. Make additional worksheets from the master provided.

LESSON FOUR

Introduction: Put a subtraction problem on the overhead version of the place value mat:

Ask the children what the "-" sign tells you to do with the materials. If they mostly respond with "take away," ask them if the sign means something else. Try to question to get "see how much bigger the top one is " or "see how much smaller the bottom one is" or something like it. Point out it can be seen either way. Do the "comparison". "There is one more TEN on the top so I'll put it below the line. There are two more ONES, so I'll put these below the line."



"What's below the line shows how much more was on the top OR how much would have to be added to the bottom to get as much as on top." Put your hand over the top material and ask the children to mentally join together

what's below the line with the bottom. Do another, having the children arrange their materials to follow you.

Do this a second way. This time say that you will use the bottom row behind the sign to indicate what to take away from the top. Do it. Then move the top down below the line. Point out the "answer" is the same in both cases.

Activity: Have the children do those worksheets using place value mats and base ten blocks. Ask them to do some by comparing and some by "take away."

Following are Masters to make ten frames and to make recording forms and worksheets to use with base ten blocks. Several such exercises are included in the pupil books.

LEVEL TWO

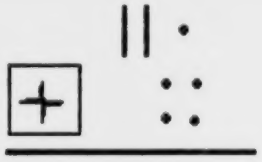
COMPUTATION: SYMBOL FORM TO MATERIAL

LESSON ONE

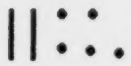
Introduction: Use the overhead projector. Write a computation for such as 21 on it. Alongside this place the base ten form:

+4

$$\begin{array}{r} 21 \\ +\ 4 \\ \hline \end{array}$$



Combine these to obtain 25 in the form:



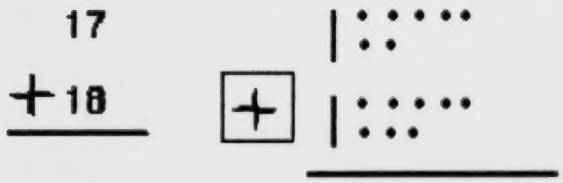
As you write the 2:

$$\begin{array}{r} 21 \\ +\ 4 \\ \hline 2 \end{array}$$

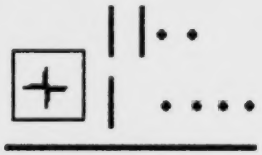
remind the children this counts 2 TENS. As you write the 5:

$$\begin{array}{r} 21 \\ +\ 4 \\ \hline 25 \end{array}$$

remind the children the 5 counts ONES. Do another that requires regrouping; i.e.:



Point out the 2 fives that make a TEN. TRADE these for a TEN, and put it in that column:



Then write the 35 to show 3 TENS and 5 ONES.

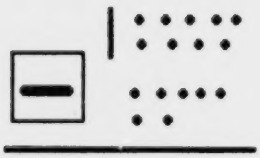
Activity: Pass out worksheets, base ten blocks and place value mats. Have the children build the numbers, then do the additions.

Watch for the regroupings. BE CERTAIN children make TENS, then TRADE for a TEN piece and put that in the right column. Always keep reminding children of looking to make a TEN from the amounts in the ONES places.

LESSON TWO

Introduction: Place a computation form like 19 on the overhead.
-7

Arrange base ten blocks next to it to represent this.

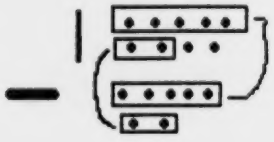


Point out that the - can show comparing or finding the difference between them or to take away the bottom number from the top one. In either case

• • or 2 results. Write that down.

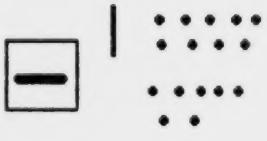
Write **1** to show the | that is there.

Manipulate the pieces to show both ways to think of the —.



What remains of the top number is

| • • or 12



Pointing to the bottom number, take away that amount from the top, again leaving

| • • or 12

Tell the children they can think of the - sign either way and that they should use the meaning they are most comfortable with.

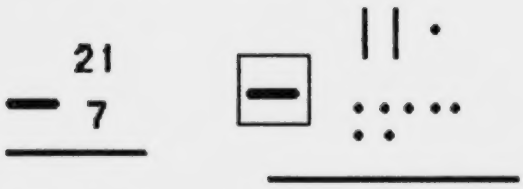
Activity: Pass out the worksheets, base ten blocks and place value mats. The children should represent EACH problem given with base ten blocks and manipulate these to find the answer. Then they write this into the computation form.

LESSON THREE

Introduction: Remind children of the counting back game, where tens had to be traded for ONES to be able to count them off the board.

Write 21 on the overhead and build it with base ten blocks alongside.

-7



Point out that there aren't enough ONES in the top number to either make a comparison or to take away ONES shown in the bottom number. Ask where the needed ONES in the top number can come from? Then TRADE one of the TENS for the ONES and place as shown:

The top number now has

more than

the bottom number, whether by comparison or by take away

Activity: Have students work on the worksheets provided using base ten blocks and place value mats.

Watch carefully for problems with trading down to obtain needed ones. Children seem to resist this much more than trading ones for tens.

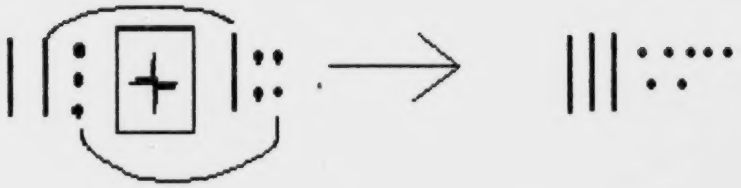
Use the same approaches when introducing computation with 3 digits after the numeration activity with 3 digits.

LESSON FOUR

Introduction: Write a horizontal computation form like 23 + 14 =

Ask the children to build this on the place value mat with base ten blocks. Observe if they change this to the vertical computation form in order to do this.

Discuss how this might be done WITHOUT changing with an illustration on the overhead projector.



Adding tens to tens and ones to ones is no more difficult this way. However, if children are more comfortable changing to the vertical form so as to keep columns aligned, let them. Do a second that involves the - sign, e.g.:

28 - 14 = or = 28 - 14

Again, build this of base ten blocks:

Again, you can subtract tens from tens and ones from ones as long as no "borrowing" is needed. Allow children to interpret - either as "take away" or as comparing, and to convert to vertical form if necessary.

<u>Activity:</u> Pass out worksheets, addition first - then subtraction. Have the children work these using base ten blocks on their place value mats.

COMPUTATION: Subtraction

Many children have difficulty with "borrowing" or decomposing tens into ones to subtract in a case like 42
- 27

Borrowing can be avoided by using an equal additions method.

Introduction: "Look at this subtraction." Write: 9
- 6

"If I add a number to both the 9 and the 6 what will be the result. Let's try 3." Write: 9 + 3 = 12
- 6 + 3 = -9

"Is the difference still the same?" (Yes, 3 is still the difference.) "If I subtract a number from both, what will be the result? Let's try 3 again." Write: 9 - 3 = 6
-6 - 3 =-3

"The difference is still 3!" "If I add or subtract the SAME NUMBER from the 2 parts, the difference remains the same. What number is easiest to subtract?" Get the idea of 0 as being the easiest across.

"Consider this" 24
- 7

"What do I add to both so what is being subtracted ends in 0?" (3)
Write: 27
- 10

"It is now easy to see the difference is 17."

"Let's try another." 52
- 36

"What do we add so the number being subtracted ends in 0?" (add 4)

"Let's try" "Adding 4 gives
56
-40

"Here is another"

$$\begin{array}{r} 50 \\ -23 \end{array}$$

"What should we add to each?" (7)

$$\begin{array}{r} 57 \\ -30 \\ \hline 27 \end{array}$$

"Practice doing subtraction this way to see if you like it better than trading a ten for ones to subtract."

COMPUTATION FORMS

Background: Children should be adept at changing a horizontal computation form (number sentence) to a vertical form and vice versa.

LESSON ONE

Introduction: Write an open number sentence on the chalkboard. Ask the children to complete it. Then write the same "fact" in vertical form and ask children to complete it. Point out that these two forms show the same number relationships. Give several such examples, then have the children complete the worksheets. Have counters available for them to use in doing this.

Examples:

Horizontal	Vertical
$2 + 3 = 5$	$\begin{array}{r} 2 \\ +\,3 \\ \hline 5 \end{array}$
$6 = 5 + 1$	$\begin{array}{r} 5 \\ +\,1 \\ \hline 6 \end{array}$
$8 + 5 = 13$	$\begin{array}{r} 8 \\ +\,5 \\ \hline 13 \end{array}$

In using vertical computation forms, children should be encouraged to "look for a ten", i.e. 2 from the 5 along with 8 make ten so 3 from 5 is left and the result if ten + three or 13. Attached are masters for forms to use to develop exercises that will have children go forth and back between horizontal number sentences, and vertical computation forms. The pupil book has several of these that are already prepared.

LESSON TWO

Introduction: Use an overhead transparency with the following "incomplete" computation forms. Arrange these so different numbers 20 through 90 can be used. The example is for 60:

2□	□8	5□	52	26
+3□	+ □6	+□7	+ □	+□8

"Which of these HAVE to be 60 or more?" Discuss the reasons for the answers.

"Which of these COULD be 60 or more?" Discuss this. Arrange similar forms for subtraction:

4□	□6	4□	42	45
-1□	- □8	-□7	- □	-□8

"Which of these HAVE to be 30 or more?" Discuss reasons.

"Which of these COULD be 30 or more?" Discuss reasons.

LEVEL TWO

COMPUTATION APPLICATIONS

Children should have base ten blocks and place value mats. Read "word" problems that have 2 digit represented numbers in them. Keep the numbers small enough so that the numbers are below thirty and the sums, where found, are below fifty. Vary the problems within the fourteen kinds that yield addition and subtraction. Read slowly enough so children can represent the quantities with base ten blocks on the place value mats. Have the children (1) find answers and (2) write number sentences for each problem. Watch for correct use of +, - , and =.

Example:
"Phyllis has twenty-three cupcakes. *(Show the cupcakes with your base ten blocks.)* She gives thirteen cupcakes to the children in her class. *(Show the sign that indicates giving away and the cupcakes with your base ten blocks).* How many cupcakes does she have left?"

Following are Masters for giving quick practice on recall of 2 part addition and 2 factor multiplication combinations, along with a sample of how the numerals should be RANDOMLY arranged on the top and sides. Use the "+" first and the "x" after work with tiles and beans and cups.

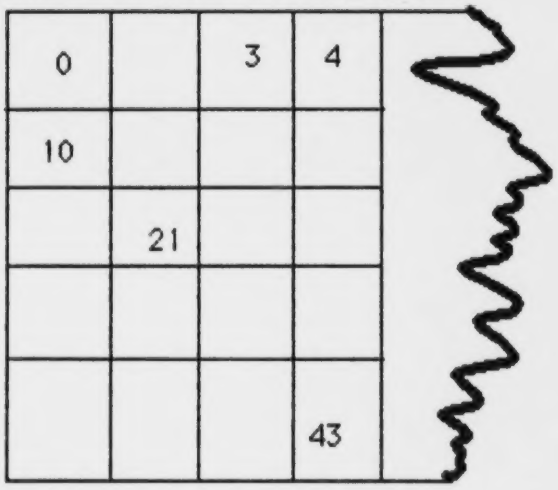
Each child should have a "+" and a "x" completed table to use for reference, except on "quick drills."

LEVEL TWO

NUMBER PATTERNS

LESSON ONE

Introduction: Use overhead transparencies of the Hundreds charts provided. Write a few numbers on the square:



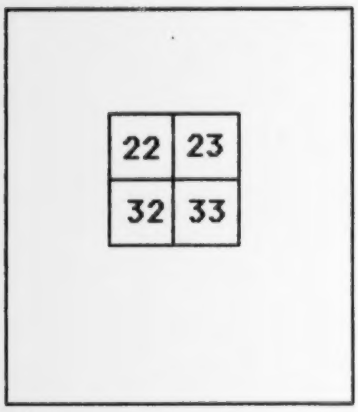
"Think carefully, and tell me where to put some other numbers into this table."

Accept responses, whether correct or not. Put the numerals where children suggest, then discuss whether this location satisfies some pattern. Which pattern?, etc. Continue until most of the matrix is filled.

Activity: Hand out the other Hundreds charts to the children and have them complete them.

LESSON TWO

Introduction: Put a completed hundreds chart - either A or B - on the overhead projecter. Using a 2 x 2 mask cover all but 4 squares on the chart. Ask the children what patterns they see.



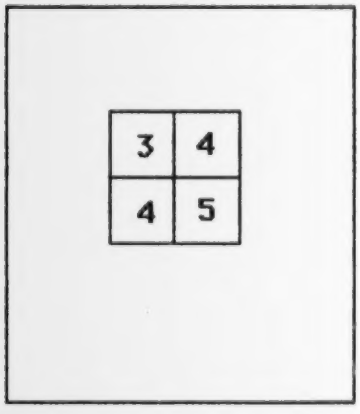
Discuss the responses. Move the mask to a different place and again have the children identify the patterns.

Extensions:

1. Use 3 x 3, 4 x 4, etc. masks

2. Use the other version of the hundreds chart

3. Use a mask on a completed addition table



4. Use a mask on a completed multiplication table.

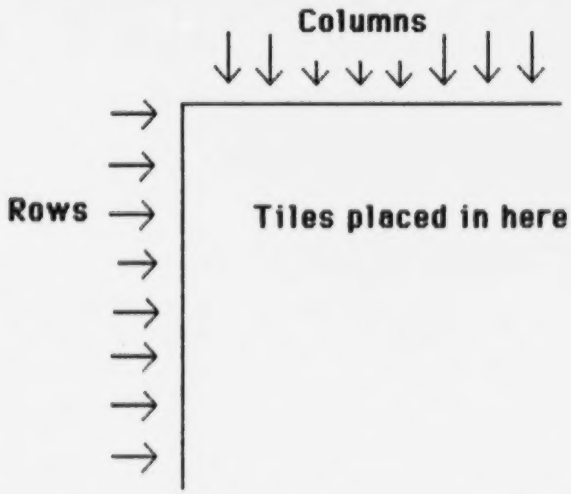
Following are Masters to use to develop templates to put over a hundreds chart to search for number patterns.

MULTIPLICATION: TILES

Background: Multiplication and division arise from the same processes of joining, separating and comparing as to addition and subtraction.

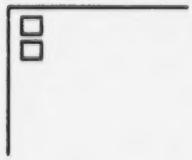
The most common use of multiplication as repeated subtraction can be emphasized along with another common application in finding areas through use of a rectangle model with tiles.

The tiles can be 1 inch ceramic, wood or heavy cardboard tiles. Needed are tiles and a workmat that has the following format:



LESSON ONE

Introduction: Place an overhead transparency version of the workmat (master supplied) on the overhead. Place two tiles as shwon and ask the children how many tiles are there. Record this (1)

	Rows	Columns	Tiles	Number Sentences
	(1) 2	1	2	2 x 1 = 2
	(2) 2	2	4	2 x 2 = 4

Then add a second column and record again (2) Point out to the children that they are to add tiles two at a time until they have 9 columns. They are then to add one tile at a time in a 3rd row until they have 9. Continue to a 4th row and so on until they have 9 rows and 9 columns.

Activity: Children should have tiles and the tile mats. As the children make arrays of rows and columns of tiles to find numbers on the table. As these are found, write them into the table.

LESSON TWO

Use an overhead transparency of the tile mat and the recording form for multiplication. Arrange 2 rows and 2 columns and show the children how this is written into the form.

Rows (x)	Columns (=)	Tiles Used	Number Sentence
2	2	4	2 x 2 = 4
2	3	6	2 x 3 = 6

Do another with 2 rows and 3 columns and repeat the process. Ask the children if they are sure of what they are to do and discuss as needed.

Activity: Pass out tiles, recording forms and work mats. The children are to complete that recording form.

Repeat with recording forms that have increasing quantities on them. Make other variations of the recording form. Examples are given.

LESSON THREE

Develop a large multiplication table into which the children can take turns entering numbers in the square:

Columns

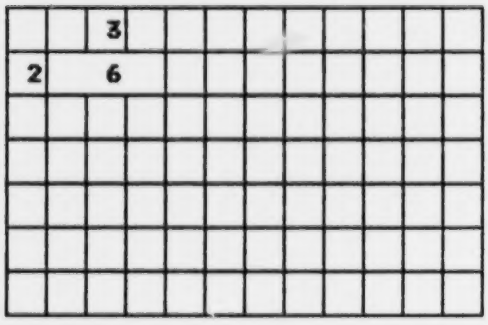
Rows

x	2	3	4	5	etc.
1					
2					
3					
4					
etc.					

Multiplication Chart

Activity: With tiles, the work mat recording forms and the multiplication table, the children are to (1) complete the recording form; (2) enter into the multiplication table; and (3) record each rectangle made on graph paper, labelling the sides and writing the number sentences:

Graph Paper



Number Sentence

2 x 3 = 6

LESSON FOUR

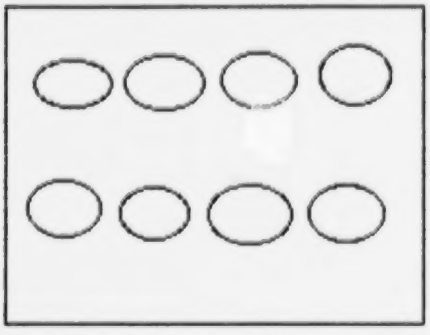
Discontinue use of the recording form and have students enter numbers directly into the table. Use some tables with numbers in them and tell the children to build these tile rectangles first.

LEVEL TWO

MULTIPLICATION: BEANS AND CUPS 1

Background: The use of beans and cups give the repeated addition interpretation of multiplication a look that emphasizes making collections as opposed to area.

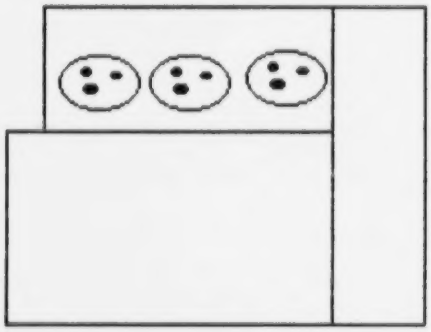
Introduction: Use an overhead projector transparency that simulates cups. By covering up with cardboard (paper blows off!) you can have as many "cups" as you need.



"I have three cups. If I put three beans in each cup, how many beans do you think I would need?" Then put three beans in each cup to show this accumulation.

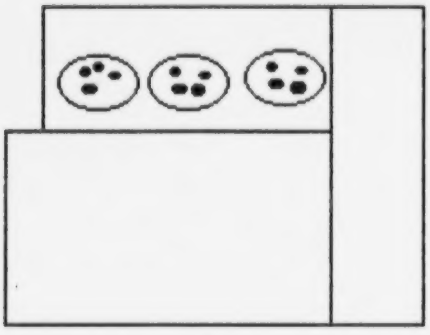
Write:

9	=	3	x	3
beans		beans in each cup		cups used



"I want to put four beans in each of these three cups. How many beans will go into each cup?" Deliberately put one bean in each "cup" until all are gone and COUNT the beans in one of them. Write the number sentence:

12	=	3	x	4
beans		cups		beans in each cup



Activity: Children should have beans, several paper or plastic cups and Worksheet One. Have them complete the worksheet. It may take several math periods.

Extensions: When children have had opportunity to see the multiplication facts up to 6x 10 with tiles and beans, then use Record Two.

LEVEL TWO

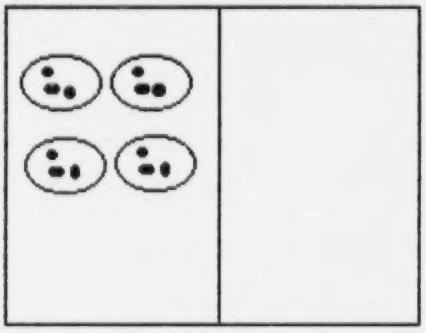
DIVISION: BEANS AND CUPS 1

Background: The tiles model gives one interpretation of division - the "measurement" or repeated subtraction interpretation. Beans and cups give opportunity to introduce both the measurement idea and the partative idea in to contrast these.

LESSON ONE

Introduction: Use the transparency mat with circle "cups" as for multiplication.

"I have twelve beans. I want to put three in each cup. How many cups will I use?" It is hoped that several children will be able to answer "four." Nevertheless deliberately distribute the beans, three at a time, into the "cups."



"How many cups would be used if I put four beans in each cup?" Again, show the placement of the groups of beans in the cups. "Were there any beans left over." "I have thirteen beans to put into cups. I need two beans in each cup. How many cups will I need?"

Some discussion may ensue with this introduction of a remainder. Point out that it is enough at this point to show the remainder in writing the number sentence:

$13 \div 2 = 6 \text{ R } 1$

Activity:
Each student should have several beans, six or seven cups, the recording forms. They are to systematically put the same number of beans into each cup, starting with two beans/cup and the number of beans increasing one at a time until six cups have been used; then three beans in each cup, etc., until six beans/cup have been used. This will take more than one class period. The worksheets using 7 beans/cup and more should be used later when place value understanding is more firmly founded.

LEVEL TWO

DIVISION: BEANS AND CUPS 2

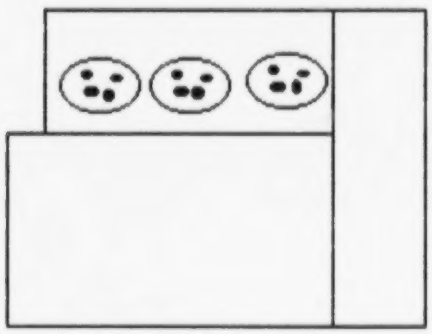
LESSON ONE

Introduction: Use the overhead in the same way as in the Beans and Cups 1 Lesson. This time, the division involves distributing in such a way as to assure equality of a given number of collections.

"I have twelve beans. I want to put these into three cups so that each cup has the same number of beans. How many beans should go into each cup?"

After any needed discussion about the responses given, carefully sort the beans one at a time into the three "cups." Write:

$$12 \div 3 = 4$$



Ask the children if you could have put the beans two at a time into the cups. Ask for any other ways to distribute the beans to be certain the cup had EQUAL numbers of beans. "I have thirteen beans to put into three cups and I want the SAME NUMBER of beans in each cup. Can I do this?" "How?"

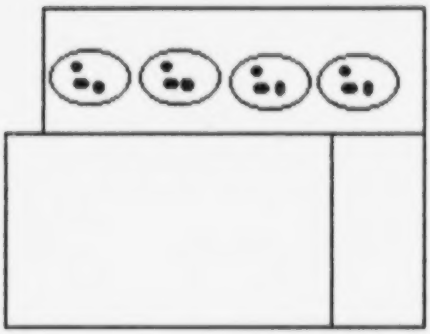
On the overhead, sort the beans so there are four in each cup again and point out the extra bean. It would disturb the EQUALITY of beans in the cup if put into any one of these so it is a remainder. Write:

$$12 \div 3 = 4 \text{ R } 1$$

"I want to distribute these same twelve beans into four cups so each will have the SAME NUMBER of beans. How many beans will be in each cup?"

Again put the beans in the overhead "cups." Write:

$$12 \div 4 = 3$$



Do one more with a different number, ie., eight beans into two cups.

Activity: The children should have beans, cups and the recording form. They are to complete these over as many class periods as needed. The + sheets can be done after Place Value. Understanding is better and children have done decode counting - ten, twenty, etc.

Extensions: Multiplication table forms can be completed by the children as they do the recording forms.

As before, keep the number of cups fixed, increasing the beans. Then one by one increase the number of cups.

Attached are Masters for recording forms and for making worksheets to be used in connection with multiplication and division using beans and cups. The pupil books has several practice exercises already prepared. The children should use beans and cups with these.w

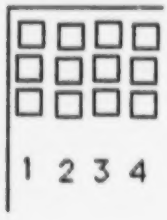
DIVISION: TILES 1

Background

Using the same models - tiles and beans and cups - for division concept development emphasizes the inverse property of these operations.

Introduction

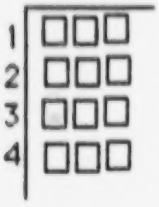
On the overhead projector, place the Tiles Workmat. Inform the students that you are going to use a given number of tiles to make rows as long as possible. "I have twelve tiles. I am going to put these into three rows. How many columns will there be?" Array the twelve tiles to verify the four columns formed:



write: $12 \div 3 = 4$

"If I used the twelve tiles to make three columns, how many rows would I get?"

Again, array the twelve tiles:



write: $12 \div 3 = 4$

"What would happen if I used thirteen tiles?"

Point out that the extra tile was not enough to make another row so it is a **Remainder.**

Write the number sentence:

$13 \div 3 = 4 \text{ R } 1$

Activity: The children should have tiles and recording forms to use. Watch to see how the number sentences are being written.

Following are Masters to be used to record work with tiles and to make worksheets to be used with tiles. Also included is a workmat sample. Note that this rectangle leads into the division "box". Students should outline on graph paper the rectangles that they make. Several exercises using these forms are in the pupil book.

LEVEL TWO

MULTIPLICATION

Introduction: Make several "two" links of UNIFIX cubes. Demonstrate linking two of these together. "How many cubes are in this link made from TWO "2" links?" "How many cubes will be in a link made of THREE "2"s links?"

On an overhead version of the recording form, show how to fill in this data.

Activity: Pair the children. Give each pair 30-40 cubes. Have them make the links and join them together as indicated on the recording form.

LEVEL TWO

DIVISION

Introduction: Show the children a ten made of UNIFIX cubes. "How many UNIFIX cubes are in this link?"

Verify by counting out loud, touching each cube in turn. "How many 'two's' in this TEN?" Verify by breaking off two links and counting aloud to find FIVE two's in the TEN.

Show the children a UNIFIX link of twelve cubes. "How should I find how many three's are in this twelve.?" Break off three's as suggested and count.

Re-attach the cubes into the link of twelve. "How should I find how many fours are in twelve?" Verify this as before.

Activity: Group the children into pairs. Give each group thirty UNIFIX cubes. Instruct them to make links having the following numbers of cubes in order: 6 7, 8, 9, 10, 11 12, 13 14, 15, 16, 17 18, 19 20, 21, 22, 23, 24, 25 26, 27, 28, 29, 30.

For each link they should see how many ways they can break the link up into EQUAL smaller links. These results should be recorded on the form provided. Once this form has been gone over and discussed, the worksheets involving base ten blocks and ten frames should be done by the students.

LEVEL TWO

WORD PROBLEMS: MULTIPLICATION & DIVISION

LESSON ONE

Introduction: Children should have beans and cups. Tell the children to use these to represent things in the problem. Read a problem slowly:

"Jean has four boxes." What can show Jean's boxes?
"There are some marbles in each box." What can show the marbles?
"There are 3 marbles in each box." Put three marbles in each box.
"How many marbles does Jean have?"

Have the children write a number sentence to show this (4 x 3 = 12).

Read additional problems that involve a repeated addition interpretation of multiplication. The children should (1) use beans and cups to show these, and (2) write related number sentences.

LESSON TWO

Introduction: Children should have several UNIFIX cubes. Read a problem:

"Fred had 15 pieces of candy." Show Fred's candy.
"He gave the same amount to each of 3 friends." Show this with UNIFIX cubes.
How many pieces did each friend get?"

LESSON THREE

Introduction: Children should have TWO colors of UNIFIX cubes. Read a problem slowly:

"Nancy has four dolls." Show Nancy's dolls with one color of UNIFIX.
"She has 3 times as many doll dresses." Show the dresses with the other color of UNIFIX cubes.
"How many doll dresses does Nancy have?"

Use the provided problems to give this kind of experience to children. Representing the objects in problems with concrete materials helps them understand the situations that require multiplication and division. Beans

and cups, UNIFIX cubes, tiles to arrange in rectangular arrays can all be used to vary the lessons.

Measurement and partition division and repeated addition and ratio comparison multiplication can be understood, with materials, at this level.

LEVEL TWO

SAMPLE PROBLEMS

A. Routine multiplication involved

1. John had six cans of tennis balls. If each can held three tennis balls, how many tennis balls did John have?
2. Francie's glove box had 6 pairs of gloves. How many gloves did Francie have?
3. There are 5 sticks of gum in a small package. Four of these packages have how many sticks of gum?
4. Hot dog buns come eight to a package. How many hot dog buns will you have if you buy three packages?
5. Halloween pumpkin stickers cost 5 cents apiece. How much do seven stickers cost?
6. Fred's classroom has only 3 rows with 5 desks in each row. How many desks are in Fred's room?
7. Crayons were on sale for $2.00 a box. How much would enough crayons for 12 children cost?
8. Toni had a cat and three kittens. She decided to knit "booties" for all of them. How many booties did she make?
9. How many tires are needed for the wheels on seven cars?
10. There are 4 boys for every 3 girls in the Center school. If the number of boys is forty, how many girls are in Center school?

B. Routine division problems:

1. Paula wished to share the twelve Big Macs she won with her two sisters. How many Big Macs did each girl get?
2. Peter sorted a collection of eighteen empty bottles into six pack containers. How many containers did he use?
3. John filled 6 glasses from a bottle containing 30 ounces of orange juice. How many ounces of orange juice went into each glass?
4. Joanne had 24 bottle caps in a collection. She sorted them equally into 4 bags. How many bottle caps were in each bag?
5. There are 20 boys on the bus. There are 4 times as many boys as girls on the bus. How many girls are on the bus?
6. Betty used 12 eggs in 3 recipes of brownies. How many eggs were used in each recipe?
7. There are six times as many pennies as nickels in my bank. I have 24 pennies. How many nickels do I have?
8. Jodie put 6 beans in each of 5 cups. How many beans did she

put into cups?

9. Tommy put 30 tiles into rows and columns to make a rectangle. He had 5 rows. How many columns did he have?

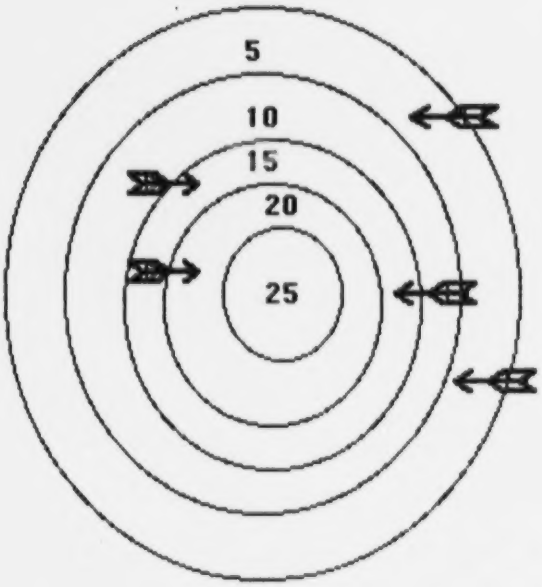
10. Tess had 29 tiles. She made 7 rows. How many tiles were in each row? How many tiles were left over?

C. Multi-step and multi-number problems:

1. John had 75 cents. A muffin cost 40 cents and a candy bar cost 38 cents. John bought a candy bar. How much money did John have left?
2. Paula, Tom and Edward collected hazelnuts. Paula found 14, Tom 27, and Edward only 10. They decided to share them. How many hazelnuts did each get?
3. Dan and his two sisters decided to play a number game in the car to pass the time. Dan wrote 3 numbers down and gave his sisters these clues:
   a. The sum of the numbers is 19
   b. They are all different
   c. Each number is less than 9

   What are the three numbers?
4. Pat, Donna, Dave and Terri exchanged Christmas gifts. How many gifts were involved?
5. Ted and Bill decided to trade baseball cards. American League cards can be traded for 2 National League cards. Tom gave Bill 12 American League cards. How many National League cards did Bill give Ted?

6. Sam threw 5 darts at the board. What is Sam's score?

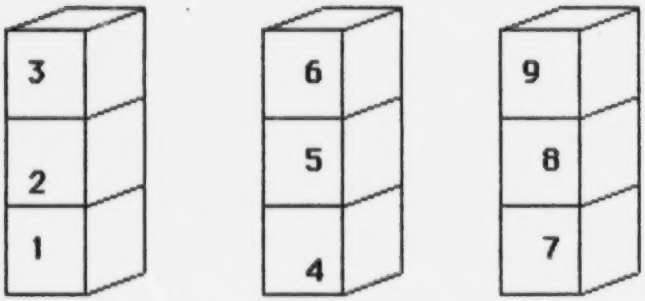


7. Winnie bought 4 stamps for 22 cents each and 4 envelopes for 10 cents each. How much did Winnie spend?

8. Frank bought 2 plastic trinkets for 8 cents each, and 3 metal emblems for 9 cents each. He gave the clerk 50 cents. How much change did Frank get?

9. Jean sold lemonade for 15 cents per glass. Down the street Fred has a sign, "Lemonade is 4 cents cheaper here!" Julia bought 6 glasses from Fred. How much did she pay?

10. Which one block should be moved to which other stack so the sum in each stack is 15?:



A B C

11. I am taller than John, but shorter than Carlo. Which of us is the tallest?

12. Pat earns 4 cents for each box of pencils she sells. She has sold 20 boxes. How many more must she sell to earn $1.00?

13. George put three coins in a soft drink machine to get a can of root beer that cost 40 cents. Which coins did he put in?

14. Pencils are sold at the school store. You can buy them at 3 for 25 cents or 9 cents each. How much can Tess ave on 24 pencils by buying them in threes?

15.

Single Cones
30 cents

Double dips	**55 cents**
Crushed nuts	**15 cents**
Marshmallow topping	**10 cents**

How much did Flo pay for a double dip cone with crushed nuts?

LEVEL TWO

AREA AND VOLUME (Geoblocks): GEOMETRY

Background: Geometry in the primary grades typically is limited to memorization of the names of common plane shapes such as circles, triangles, rectangles, etc.

These plane shapes ALWAYS are found in the concrete as faces of shapes that occupy space, so the plane shapes should be introduced as faces of three dimensional shapes. The lessons that suggest the use of geoblocks do these.

LESSON ONE

Introduction: Show the children several of the geoblocks. Point out how the faces are in the form of different known shapes like triangles, squares and non-square rectangles. Also point out the SAME shape can be found on two or more blocks by matching the faces.

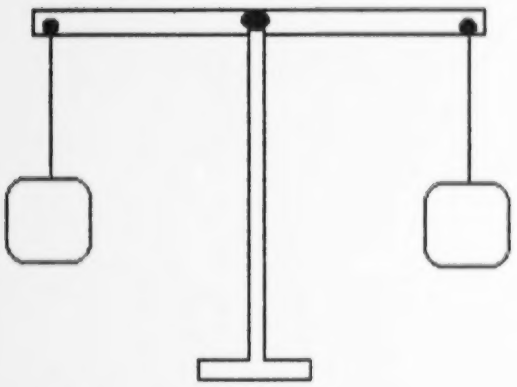
Activity: Give each child a set of 4 or 5 DIFFERENT geoblocks. Have each child find as many faces that are ALIKE as they can. Have them trace these ALIKE faces on the same piece of paper. Then have them find as many DIFFERENT rectangles as they can and trace these on the same sheet of paper. Do the same for triangles and squares. Have the children compare drawings to see how many of the SAME shapes they have traced. Hang up all of the "rectangle" sheets and have the class determine how *many* DIFFERENT rectangles were traced. Do the same for the "triangle" sheets and the "square" sheets.

LESSON TWO

Ask the children to describe orally as many ways in which the blocks can be ALIKE or DIFFERENT as they can. List these on the board and discuss. Some examples are number of faces, the number of edges, the number of "corners" (vertices) on the shapes of the faces, by having the same shape on them.

LESSON THREE

Introduction: Show the children how a primer balance works. If you don't have these, make some balances by hanging milk carton buckets from a beam suspended at its center.

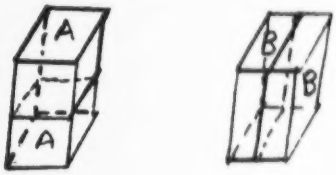


Hold up two unlike geoblocks. Ask the children which they think will "tip the balance." Try balancing them. If one side goes down, discuss why this happens. If they balance, point out they have DIFFERENT shapes, but WEIGH the same.

Activity: Provide pairs or trios of children with a balance and 3 or 4 geoblocks. Ask them to use the balance to order these from lightest to heaviest.

LESSON FOUR

Introduction: Show the children how two of the same blocks can be put together to make a larger block in more than one way.



Ask the children how A and B compare in the space they occupy. Emphasize concepts like: "Each is HALF of the same block." Show how this is different from using two DIFFERENT blocks to make the same shape in more than one way.

Activity: Give pairs of children 7 or 8 geoblocks with at least one case of 2 alike. Have them compare these:

"Which is the largest? How do you know?"
"Are any the SAME volume? How do you know?"
"Can you find cases where two of your blocks fitted together make the same shape as another of your blocks?"
"What can you find out about those blocks?"
"How are space occupied (volume) and weight different?"
"Do balances measure weight or volume?"
"How might you find out how much space the blocks occupy?"

LEVEL TWO

GEOMETRY: GEOBLOCKS

LESSON ONE

Make "geojackets" of single blocks by tracing faces of geoblocks on a sheet of paper. Three or four of these will fit on one 8 1/2 x 11 sheet. Organize teams of three children. Each team should have three or four of the geojacket sheets. Have them find all of the geoblocks in a collection of 20-25 that have a face that fits in the "geojacket." Have them record this number. Vary the sheets among the groups so the lesson can be repeated a few times.

LESSON TWO

Make geojackets using two blocks placed side by side so that a single outline shape has faces from two blocks in it. Repeat the procedures in Lesson One.

LEVEL TWO

GEOMETRY: PATTERN BLOCKS

LESSON ONE

Introduction: Use overhead transparency Pattern Blocks. Show the children how a small set of them - 2 reds, 5 greens and 2 blues as an example - can be placed into a variety of arrangements. Suggest making pattern block "houses," "horses," etc.

Activity: Give children each a small set of Pattern Blocks to make several shapes from. Have them trace the outline of the shapes made. These then can be "problems" for other children to solve.

LESSON TWO
Two sheets are furnished that give examples of pattern block activities. Use a Pattern Block template to create similar activity sheets.

1

Following are several samples of commercial worksheets to be used with Tangrams and Pattern Blocks. The other recording form is in the student's book. It should be made into a large chart to hang in the room on which children can paste drawings of the shapes they have succeeded in making.

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[1] *PATTERN BLOCK PROBLEMS FOR PRIMARY PEOPLE and LET'S PATTERN BLOCK IT have several similar kinds of activities for this age group.*

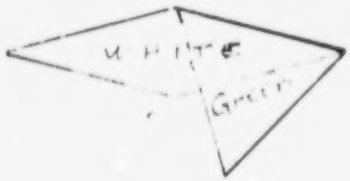
LEVEL TWO

GEOMETRY: PATTERN BLOCKS 2

On the overhead projector, place one of each Pattern Block shape. Use overhead transparency versions of the Pattern Blocks. Place a green triangle so its angle coincides with one on the blue "diamond."

Tell the children these angles are equal because one matches the other.

Place the white (or grey) "diamond" on the green triangle so the smaller angle of the white is on an angle of the green.



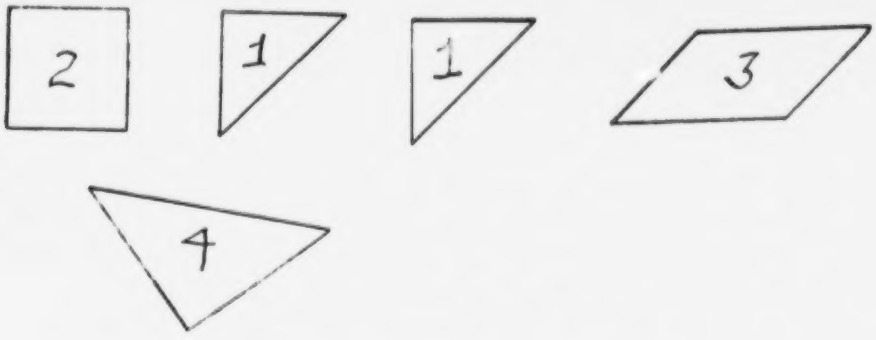
Ask the children what part the white angle is of the green angle. Work this to get them to see it is ONE HALF of the green angle. Pass out the worksheet to be done by groups of three children, each group with a collection of Pattern Blocks that includes two of each piece.

LEVEL TWO

AREA (Tangrams): GEOMETRY 2

LESSON ONE

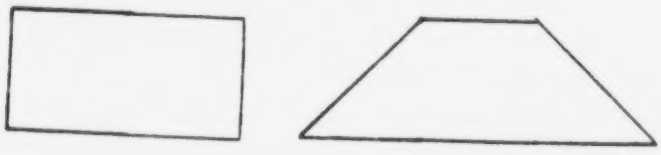
Introduction: On the overhead projector, use 2, 3 or 4 of the five smallest tangram pieces to make some different shapes:



Some examples are given:

1. From the two #1, make #2, #3 and #4. Ask the children what this tells you about the AREA covered by #2, #3 and #4.

2. With two #1 and #2, make:



Again ask the children what is true of the areas covered by the TRAPEZOID and the RECTANGLE.

LEVEL TWO

GEOMETRY: ANGLES

LESSON ONE

Introduction: Use the overhead projector. Place a green piece on top of a blue piece and poin.t out how the angles match. (An angle of the green with the smaller angle on the blue).

Place a second green piece on top of the blue and point out how TWO angles of the green match ONE large angle on the blue.

"The angle on a green is equal to a smaller angle on the blue."

"The angle on a green is half of a larger angle on the blue."

Label the angles as follows and leave on the board or somewhere as a reference for the children.

Activity: Have the children find all ways that angles can be put together to make larger angles.

An example is: $2A = B$, $A + C = D$

Some suggestions to give the children:

"How many **B** angles make a **D** angle?"

"Can you make and **E** angle from just **B** angles?"

"Arrange the angles from smallest to largest."

"Find the angles on the yellow piece."

"Can the **C** angle be made, using smaller angles?"

Have the children record all such angle relationships that they find.

Activity: Give each pair of children these five pieces of a tangram set and have them put them together 2, 3, or 4 at a time to make different shapes. Have them trace around the shapes they make. Display these sheets so all can see the variety of shapes it is possible to make. Pass out the lower numbers half of the worksheets and have the children cover the shapes on them with pieces selected from the five pieces given them.

LESSON TWO

Each pair of children should again have the five smallest tangram pieces. Have them cover the outlines of the top numbered half of the worksheets using ALL FIVE of the tangrams.

LESSON THREE

Using all seven tangrams, the children are to make their own shapes from them. They should trace around these and these then become "problems" for other children to work on.

An alternative is to use the sheet made up of the smallest right triangle. The children are to color in SIXTEEN of them so that they are all connected (an example is given.) These become outlines for other children to fill in with tangrams. Not all of them will be possible so have two boxes. The children can put those they are able to complete into one and those still not solved into another.

LESSON FOUR

Pair the children. Each pair should have a set of tangrams. Have them do the following:

1. Find sides that match on the pieces.
2. Find all of the different lengths of sides on the pieces.
3. Which of the pieces cover the same amount of paper?
4. Which angles match on the pieces?

LESSON FIVE

Introduction: After children have had an introduction to tangram work with all seven pieces, this continuing activity can be used.

Prepare a transparency from the provided table. Make a tagboard table with roughly 8" x 8" cells with appropriate headings. Tell the children this will

hang in the room. When a shape is made that can be placed on one of the cells, they are to trace that shape and pin the tracing in that cell.

Each child should have a set of tangrams and a recording table. Refer to the transparency on the overhead by pointing to the cell for (insert square picture here) and 1 piece. Ask the children to find one piece shaped like that and hold it up. Put your (insert square picture here) in that cell on the transparency and have the children write **"yes"** on their tables.

"Can you make a square from 2 pieces?"

Have a child come up and arrange two pieces to make a square on the overhead. The children should write **"yes"** in that cell and in EVERY cell where they are able to make the required shape. Tell the children the tagboard table will be up all year and they are to put the tracings and drawings of all shapes they make on the tagboard table as well as write **"yes"** on their tables.

1

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[1] *TANGRAM PATTERNS is a good source of tangram problems for this age group.*

LEVEL TWO

GEOMETRY: TANGRAMS

LESSON

Pair the children. Each pair should have a set of tangrams. Have them do the following:

1. Find sides that match on the pieces.

2. Find all of the different lengths of sides on the pieces.

3. Which of the pieces cover the same amount of paper.

4. Which angles match on the pieces?

LEVEL TWO

GEOMETRY: GEOBOARDS

LESSON ONE

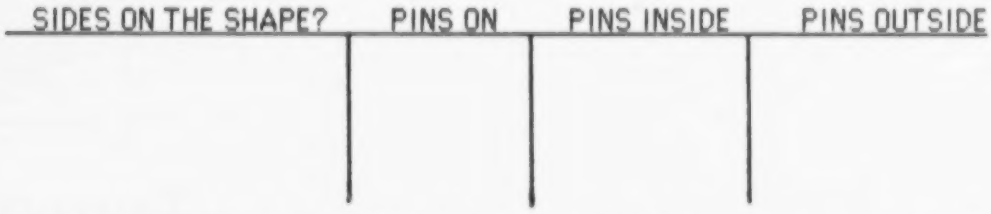
Introduction: Show the geoboards to the children. Stretch a band to make a shape. Point out the band TOUCHES some pins - these are ON the shape. Point out the pins INSIDE the shape and the pins OUTSIDE the shape.

Activity: Children should have geoboards and dotpaper. Have the children build a 3 sided shape. Ask the children who have 1 pin INSIDE the shape to hold these up. Then ask the children who have 2 pins INSIDE the shape to hold these up. Continue until all have been displayed. Repeat until they all can identify INSIDE pins. Have them make a 4 sided shape. This time ask for 1, 2, etc. pins OUTSIDE the shape. Again discuss.

Have the children build a different 4 sided shape. Ask for those whose shapes touch 4, 5, etc. pins. Discuss these cases until children seem to know when pins are ON (or touched) INSIDE and OUTSIDE.

LESSON TWO

Introduction: Make an overhead transparency of the recording form:



Make shape, show it to the children and record the correct numbers on the transparency chart. Do a second. Have the children build a shape on their geoboards. Record several of their results on the chart.

Activity: Pass out the recording forms and have the children each make shapes to complete ALL lines of the form. Classroom challenges:

"Make a shape so there are AS MANY pins inside as pins outside.

"Make a shape with half as many pins inside as outside.
Following are the recording forms provided in the pupil's book for these lessons.

LEVEL TWO

USING LOGO

If you have LOGO available to your students, here are suggested primitive commands for them to explore that are appropriate at this level. Have a poster in the room with these listed and defined. Encourage the students to keep a "dictionary" of these commands.

ST SHOW TURTLE This brings the "turtle" at the HOME position on the screen. Without this, the commands to move the turtle are of little value.

CS CLEAR SCREEN This clears all "turtle trails" from the screen and puts the turtle at the HOME position. It is to be used when children want to have a "clean paper" to write on.

HOME This command sends the turtle to the HOME position in the center of the screen, leaving a trail as it goes if the pen is down. The pen should be down at all times for most children of this age.

FD FORWARD This sends the turtle a given distance in the direction in which it is headed. Thus, FD 50 will move the turtle ahead 50 units.

BK BACK This backs the turtle up a given distance in the direction in which it is headed. It is the inverse of FD - like "-" is the inverse of "+".

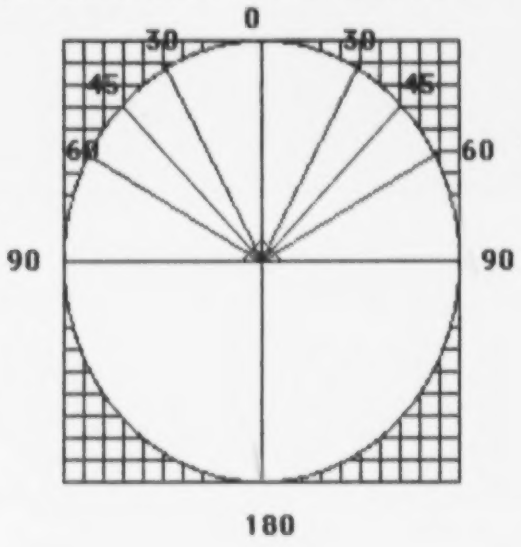
RT RIGHT There are a total of 360 units of turning when a turtle turns through a complete circle. Hence, RT 90 will turn the turtle CLOCKWISE through 90 degrees or a quarter turn. FD 180 points the turtle in the opposite direction.

LT LEFT This is the inverse in that the turtle turns COUNTER CLOCKWISE a given number of degrees of a circle.

All of the primitive commands that result in motion of the turtle, (RT, LT, FD, BK) should be preceded by having the children move their bodies in response to command as the turtle would move. You should also use walnut shells or cardboard or wood "turtles" and move these on an overhead transparency grid in response to commands. Some sample "off computer" lessons and some sample introductory lessons:

LESSON ONE

Introduction: Make a large circle to put on the floor. Use butcher paper or oilcloth or some similar material:



Activity: Each child in turn should be the "turtle". Give the "turtle" commands such as:

FORWARD 5
FORWARD 10
BACK 10
BACK 20
RIGHT 30, etc.

Children should point their feet along the line that leads to 30 on the circle perimeter.

RIGHT 45
LEFT 30
RIGHT 90
LEFT 60, etc.

Keep at this until children can ALL follow these commands.

LESSON TWO

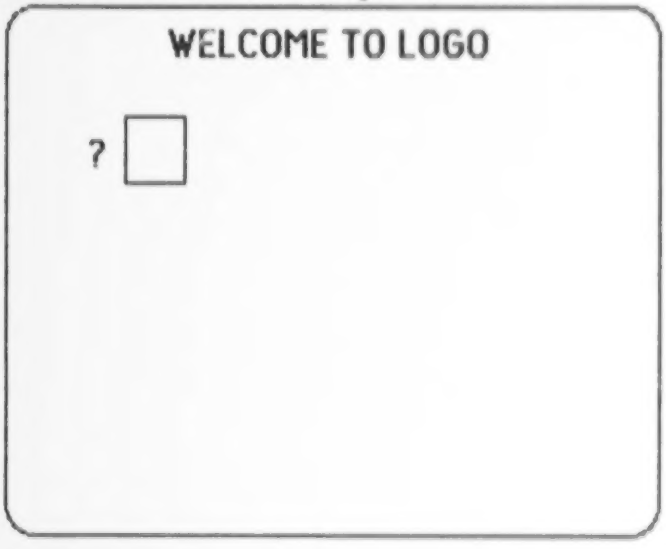
Introduction: Make a transparency similar to the floor mat (a master is provided). Move the turtle on this transparency to show FD, BK, RT and LT moves much as in LESSON ONE.

Be sure you make it clear that RT and LT only TURN the turtle. After turning, the turtle must be told to move FD or BK. Show this by RT 30, FD 20 and similar combined commands.

Activity: Have the children draw lines on the worksheet to show "turtle" moves. Watch for children's interpretations of the combined command moves.

LESSON THREE: AT THE COMPUTER

Load LOGO so the following is on the screen:



"Type in ST"

Point out the location of the turtle in the center of the screen.

"The turtle is now in its HOME position."

"Type in FD 20."

"The turtle has now taken 20 steps ahead."
"Type in HOME."

"We are putting the turtle back where it started so we can see how long a trail it left."

"Type in RT 30."

"We are turning the turtle so it can go ahead in a different direction."

"Type in FD 40."

"Type in HOME."

"Compare the turtle's trail the second time with the first."

"Type in RT 90."

"Type in FD 60."

"Notice the turtle's trail is longer when the number is bigger."

"Does RT 90 affect the length of the turtle trail?"

"Type in CS."

Now the turtle is HOME and all turtle trails are gone.

Have the children use the FD , RT , HOME and CS commands to explore how to make the turtle move around and leave trails of different kinds.

LESSON FOUR:

Follow the same procedures using BK and LT

"Which command makes the turtle move like the hands of a clock?"

"Which command reverses the direction of the turtle and moves it back along its trail?"

"How do you get rid of all of the turtle trails?"

"How do you get the turtle to go HOME?"

"Does it leave a trail when it goes HOME?"

Devise additional activities involving these primitive commands using ideas from the following good sources:

"The Turtle's Source-Book", Bearden, Martin & Muller. Reston Publishing Co.

"Learning With LOGO", D. Watt. McGraw Hill.

"LOGOWORLDS", Heller, Martin and Wright. Computer Science Press.

"Exploring with LOGO I", Allyn & Bacon Co.

"LOGO Discoveries," Series, Creative Publications 2.

BLACK LINE MASTERS

for many of the

WORKSHEETS

and

RECORDING FORMS

MATHEMATICIAN: _____

My Bean Number Sentence

MATHEMATICIAN:_____

My Unifix Number Sentence

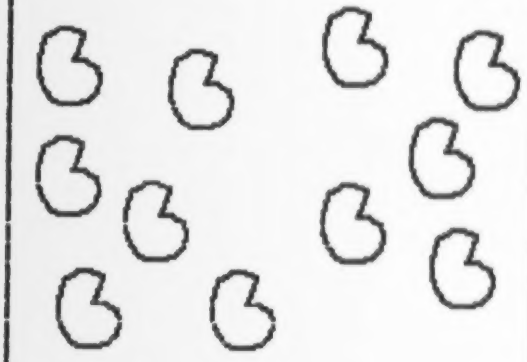
Mathematician:_____

My Unifix Record

NUMBER	NUMBER SENTENCES

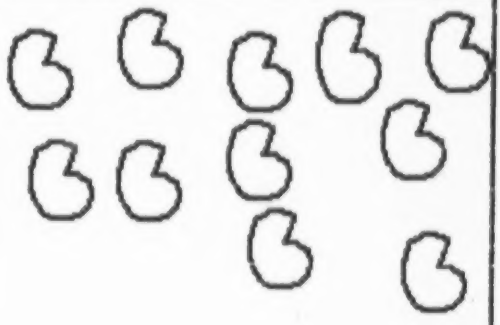
MATHEMATICIAN:_____

My Bean Record



___ + ___ = 11

11 = ___ + ___



___ + ___ = 11

11 = ___ + ___

___ + ___ = 11

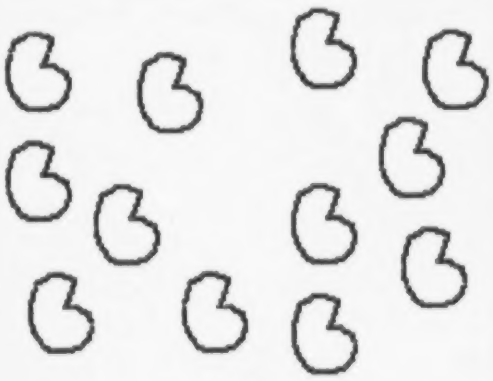
11 = ___ + ___

___ + ___ = 11

11 = ___ + ___

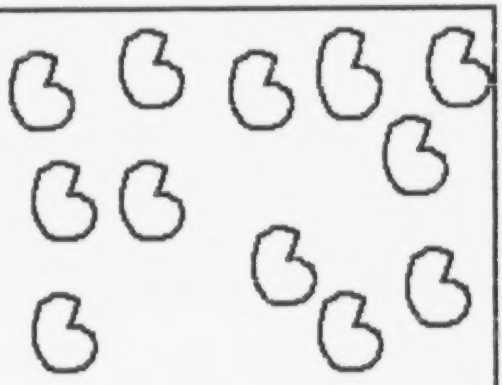
MATHEMATICIAN:_____

My Bean Record



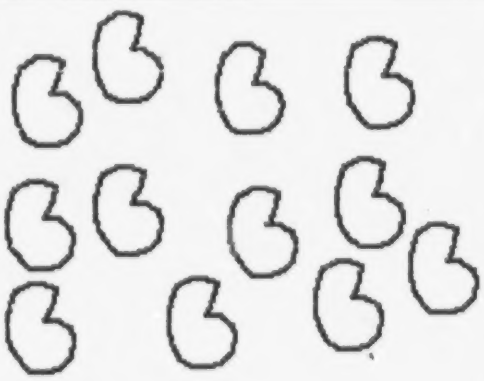
___ + ___ = 12

12 = ___ + ___



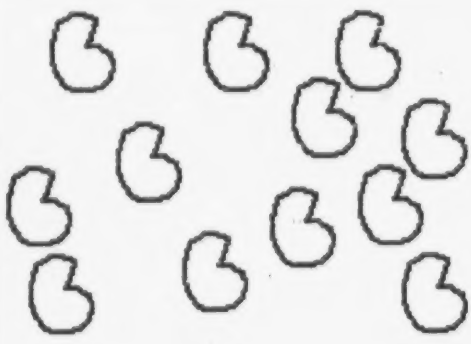
___ + ___ = 12

12 = ___ + ___



___ + ___ = 12

12 = ___ + ___

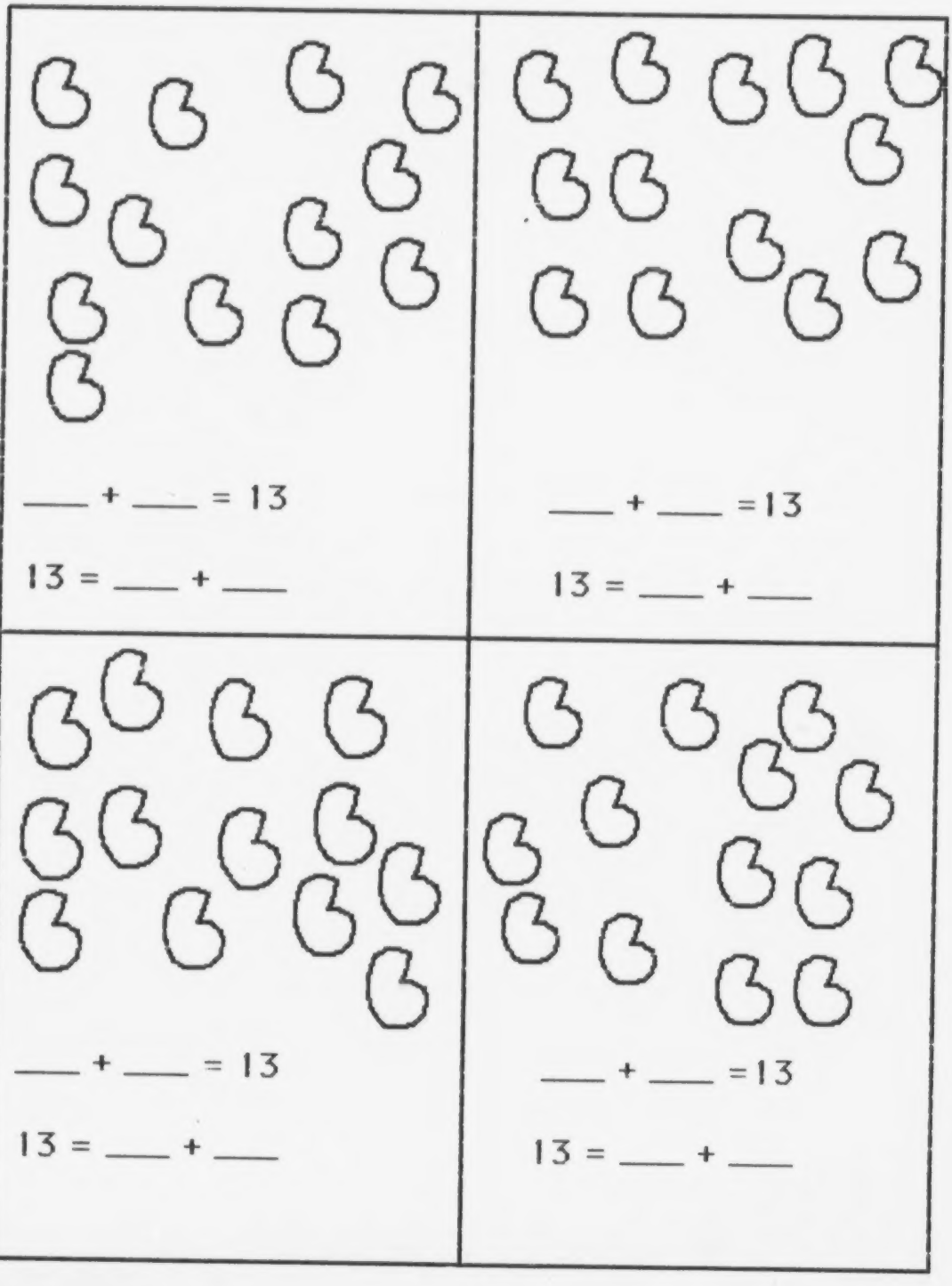


___ + ___ = 12

12 = ___ + ___

MATHEMATICIAN:_____

My Bean Record



___ + ___ = 13

13 = ___ + ___

___ + ___ =13

13 = ___ + ___

___ + ___ = 13

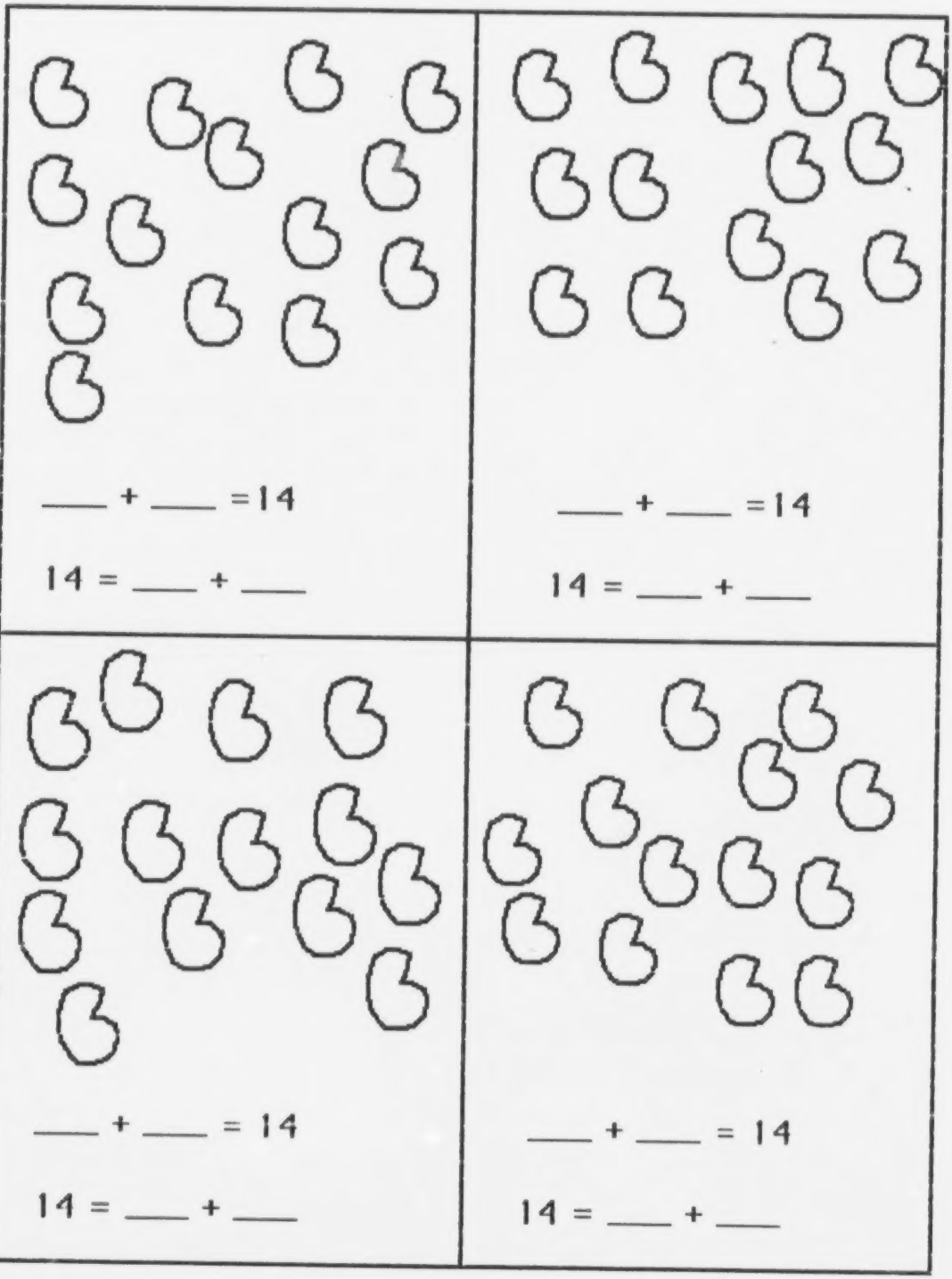
13 = ___ + ___

___ + ___ =13

13 = ___ + ___

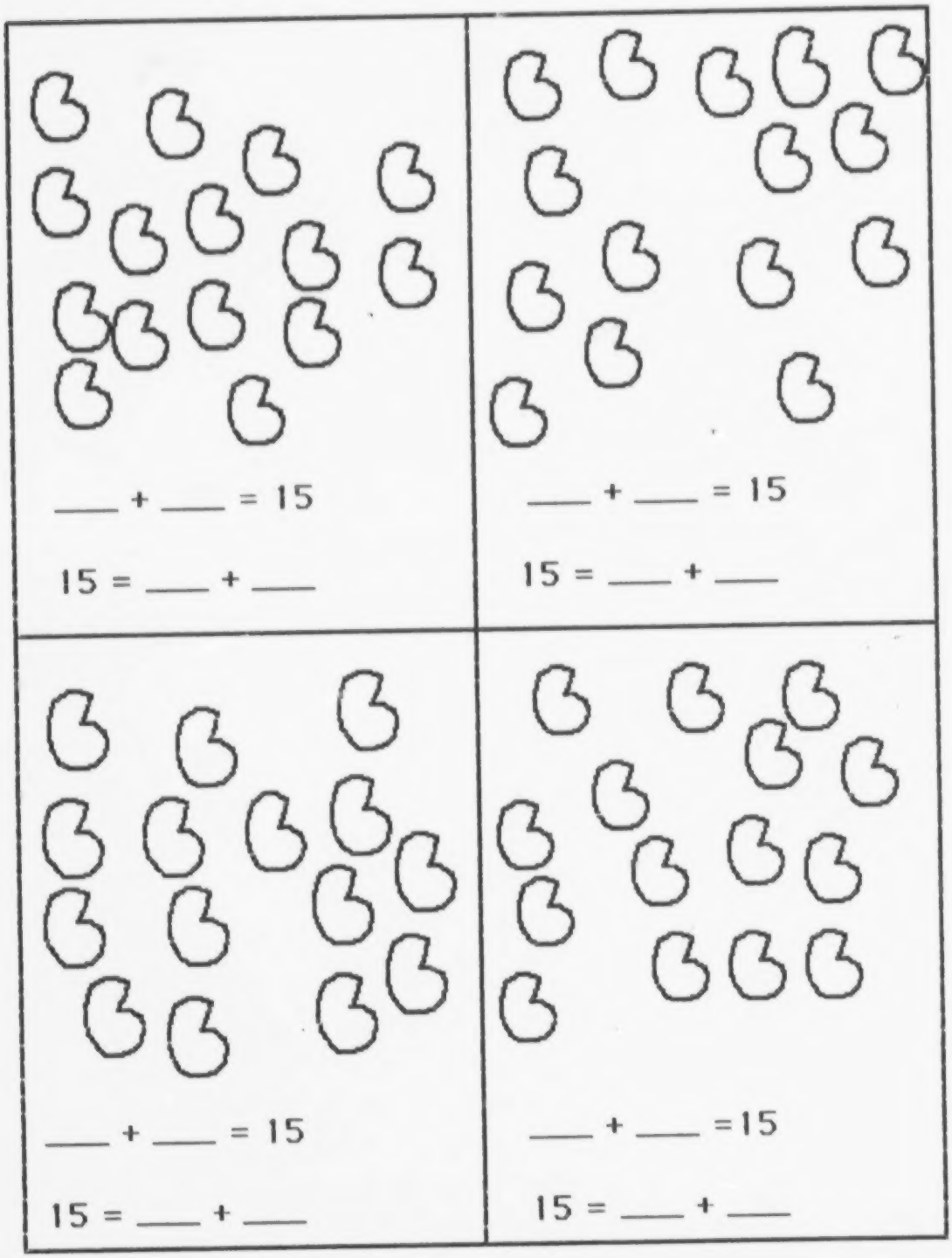
MATHEMATICIAN:_____

My Bean Record



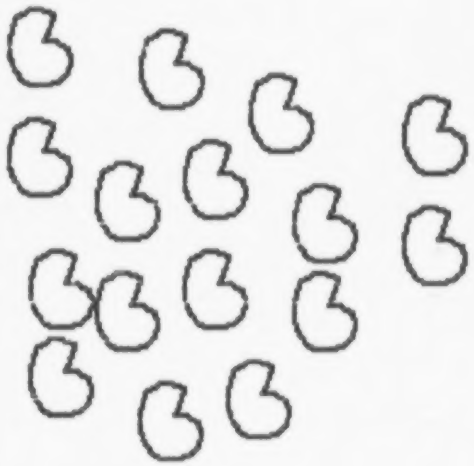
MATHEMATICIAN:_____

My Bean Record



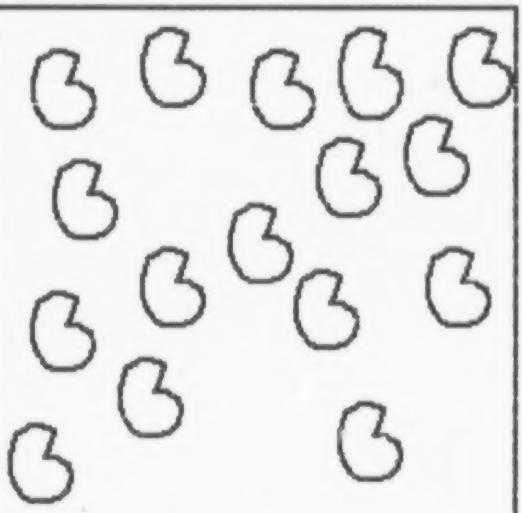
MATHEMATICIAN:_____

My Bean Record



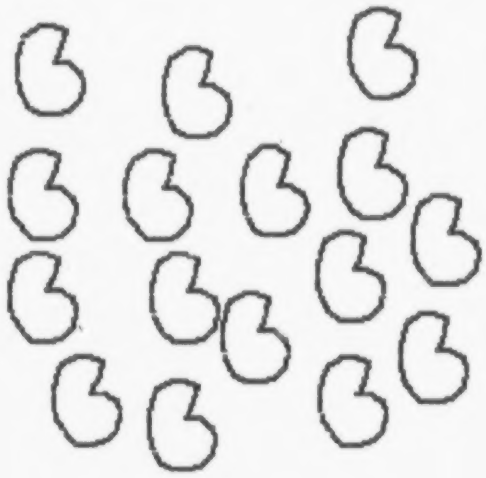
___ + ___ = 16

16 = ___ + ___



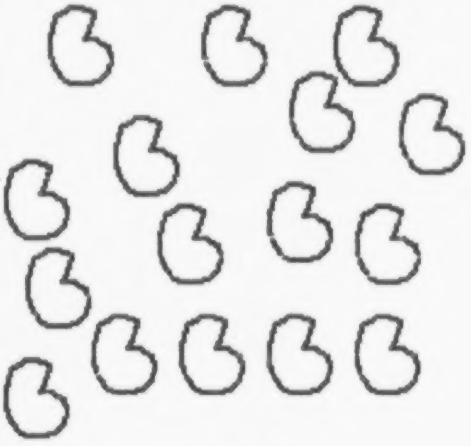
___ + ___ = 16

___ + ___ = 16



___ + ___ = 16

16 = ___ + ___

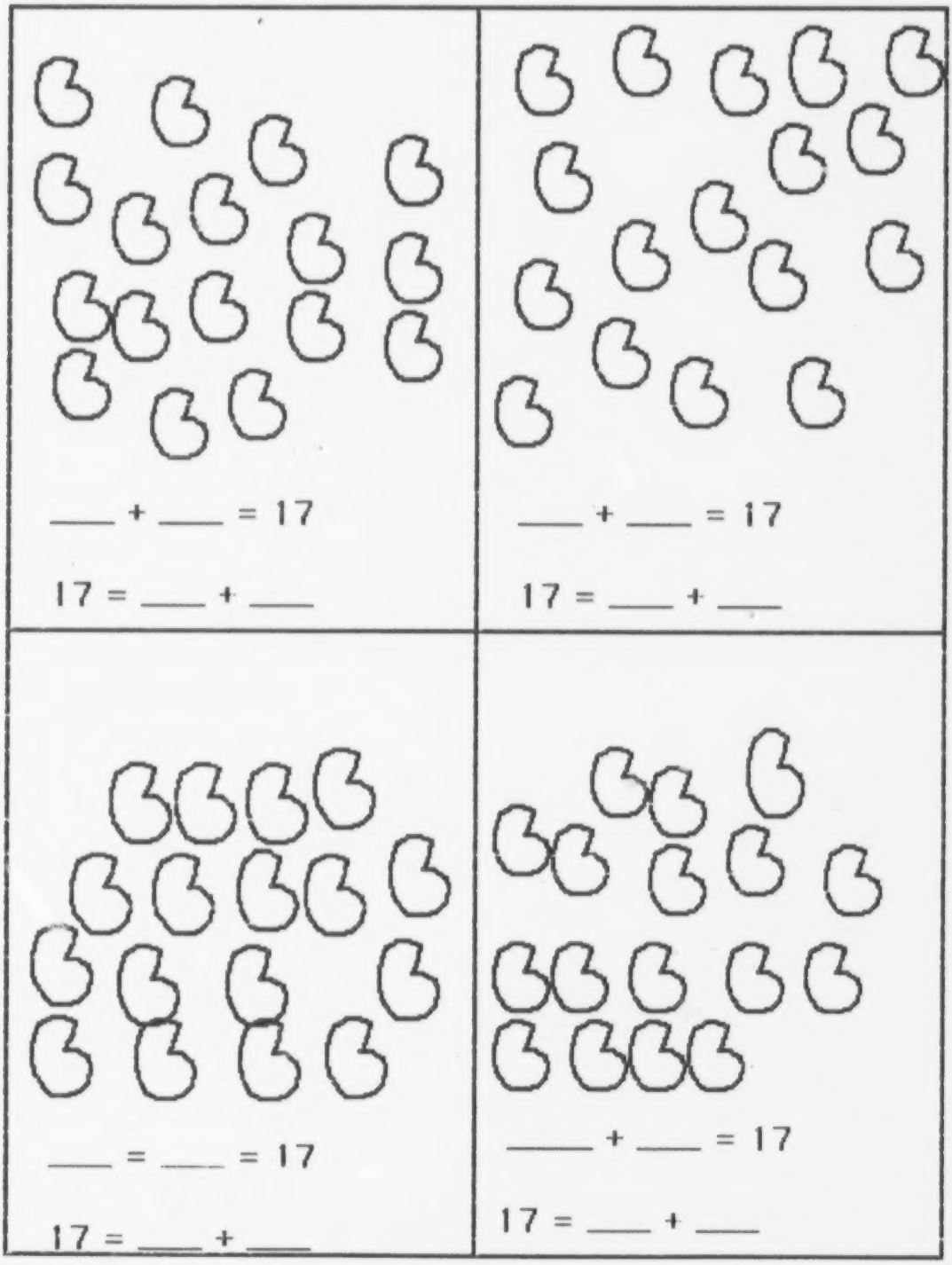


___ + ___ = 16

16 = ___ + ___

MATHEMATICIAN:_____

My Bean Record



___ + ___ = 17

17 = ___ + ___

___ + ___ = 17

17 = ___ + ___

___ = ___ = 17

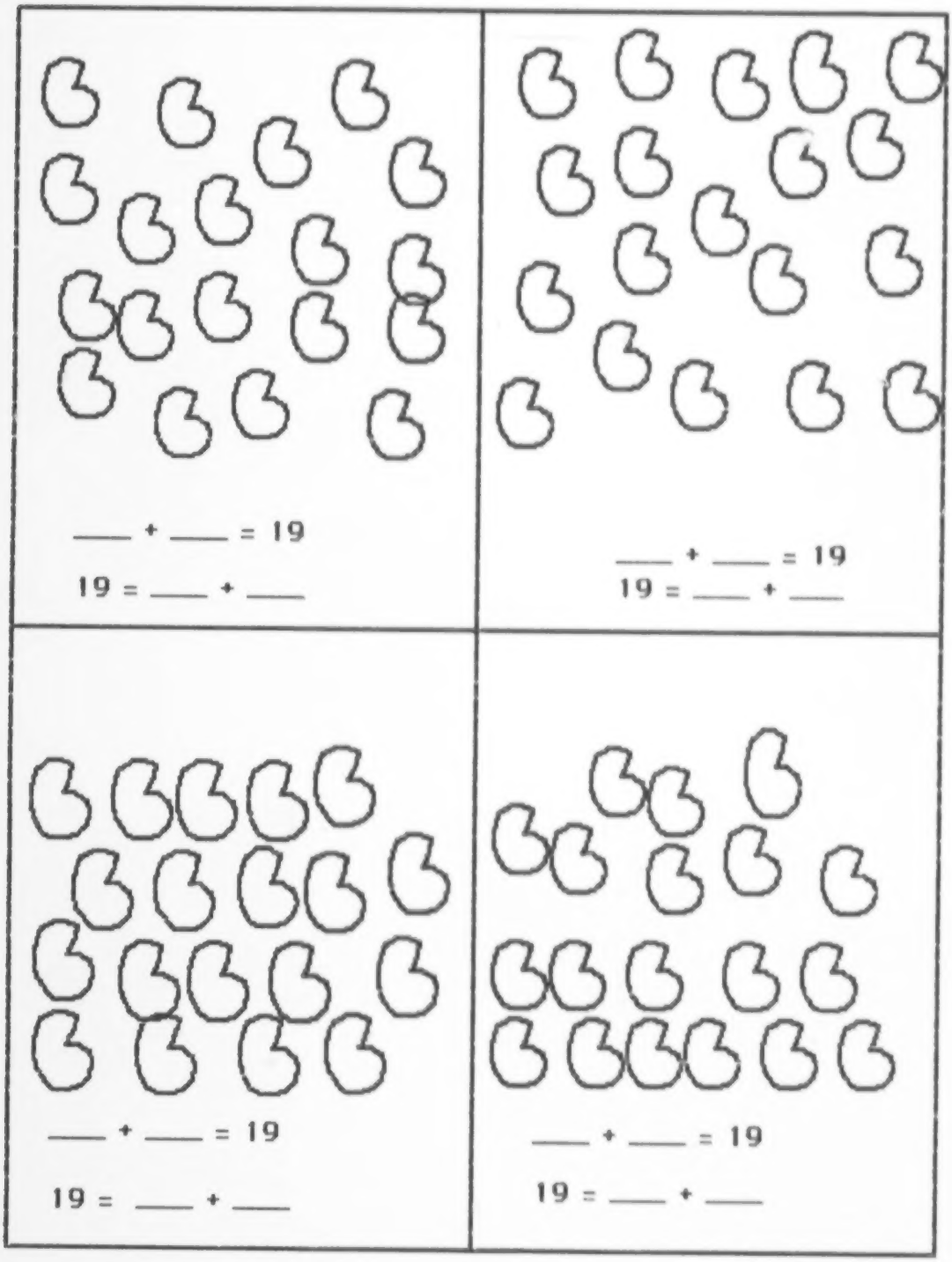
17 = ___ + ___

___ + ___ = 17

17 = ___ + ___

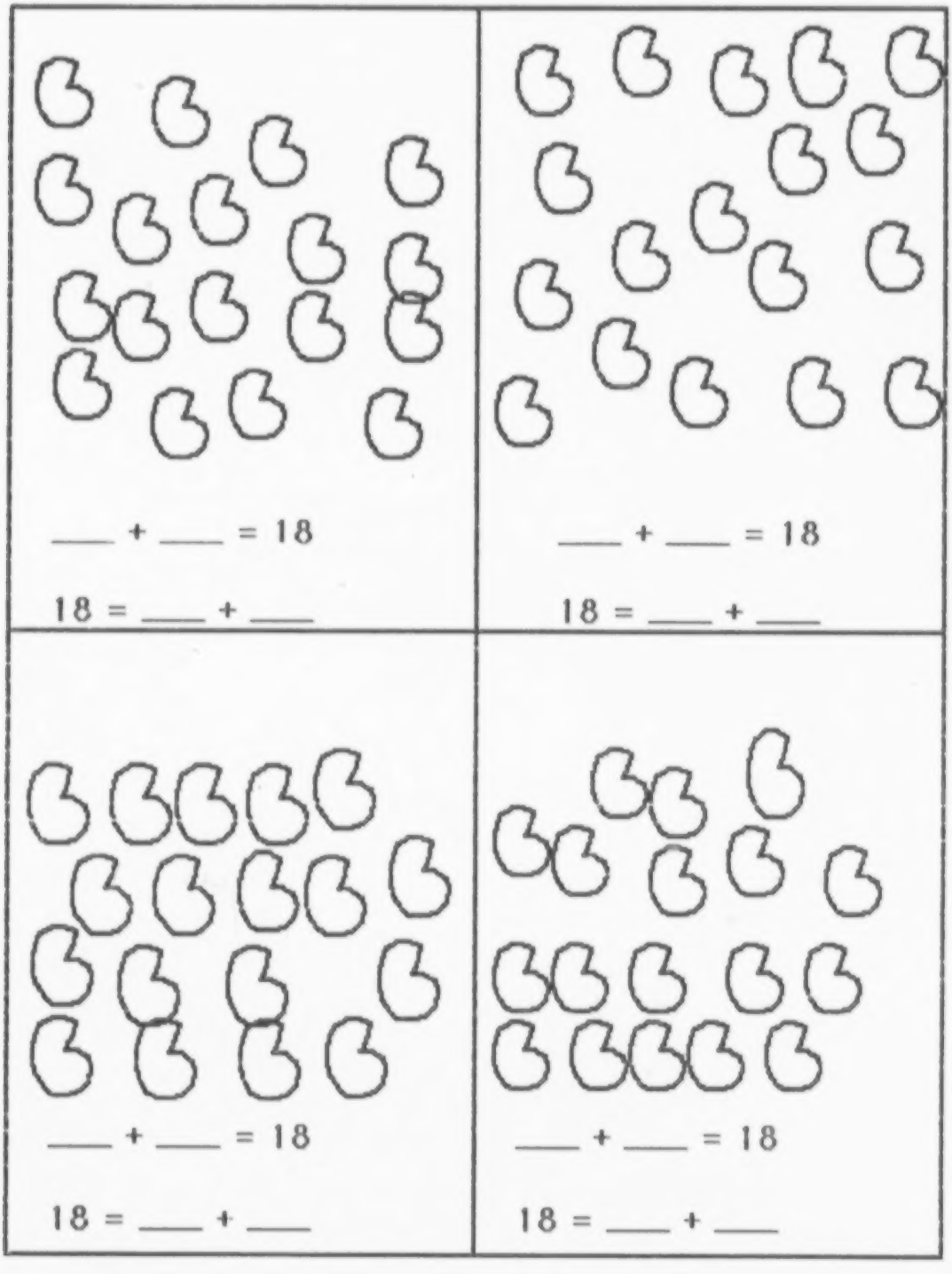
MATHEMATICIAN:_____

My Bean Record



MATHEMATICIAN:_____

My Bean Record



___ + ___ = 18

18 = ___ + ___

___ + ___ = 18

18 = ___ + ___

___ + ___ = 18

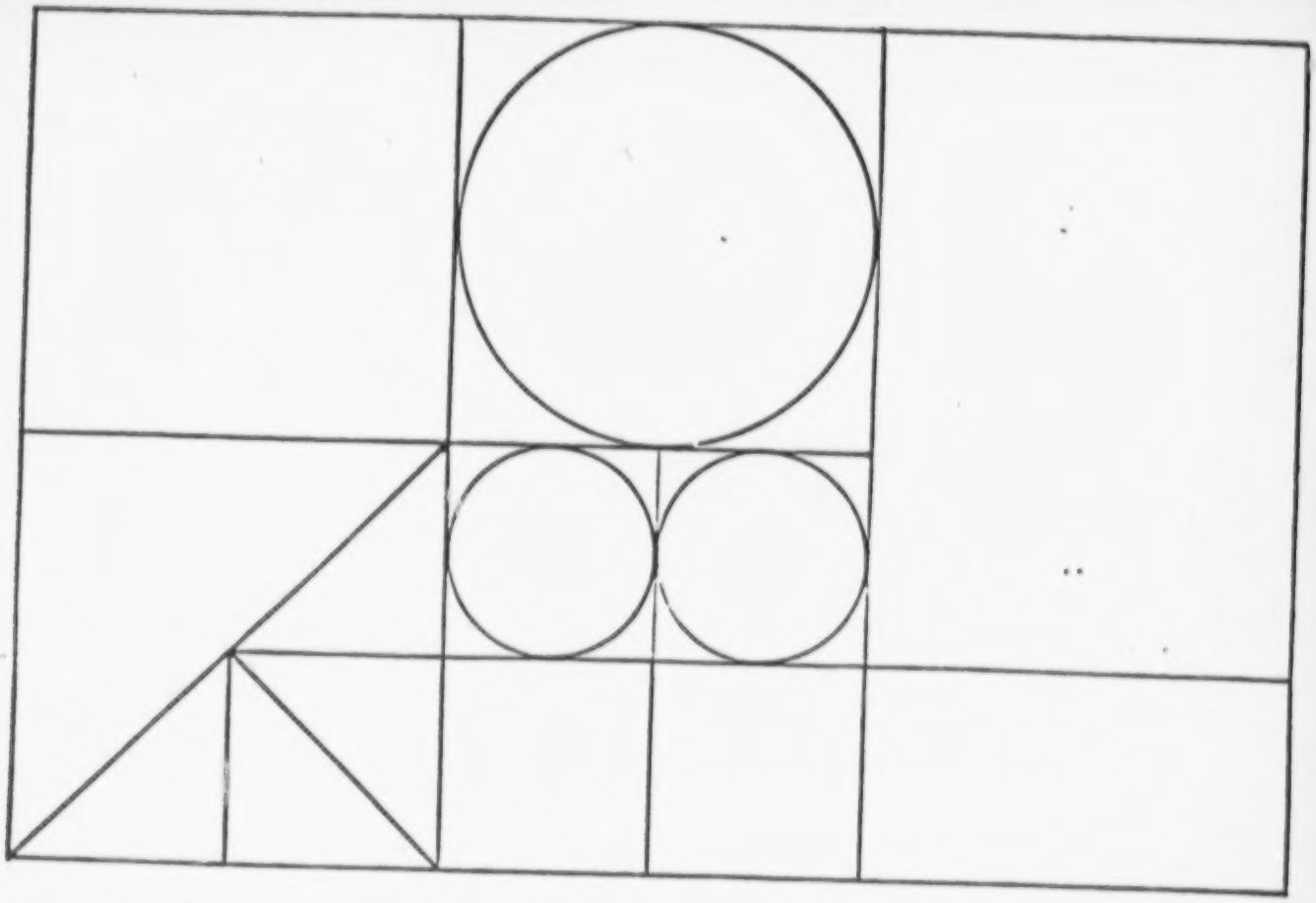
18 = ___ + ___

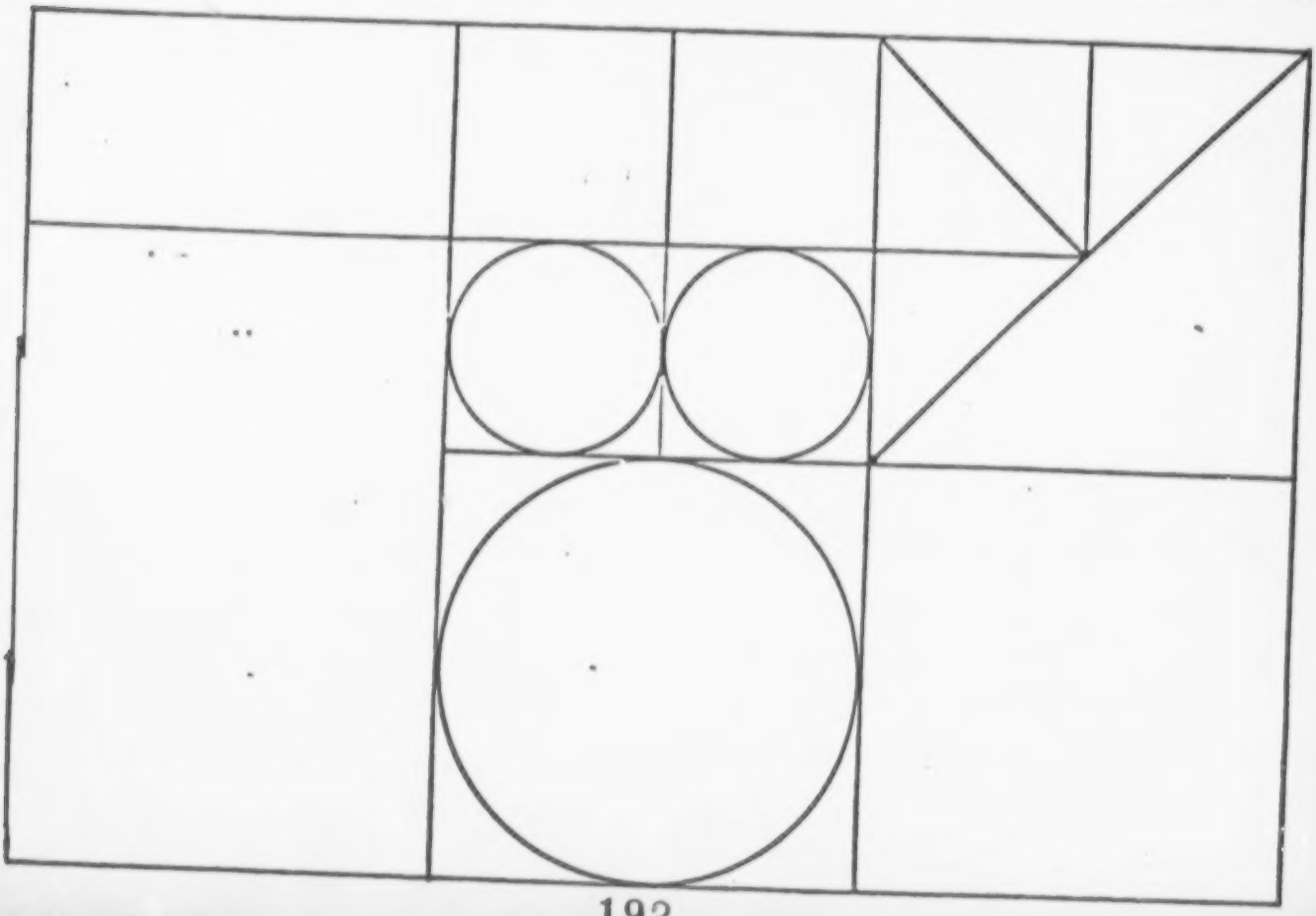
___ + ___ = 18

18 = ___ + ___

PATTERN TILE WORKSHEET

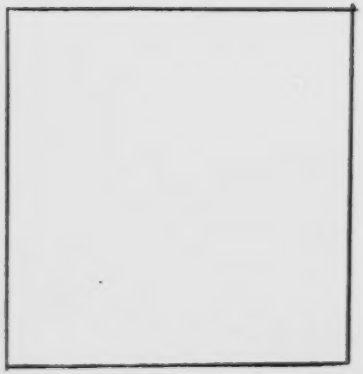
Here are the first three in tile patterns. Make the next two and record these by coloring in squares on your form:





MATHEMATICIAN:_____

PUT TOGETHER GAME
WORKMAT



=

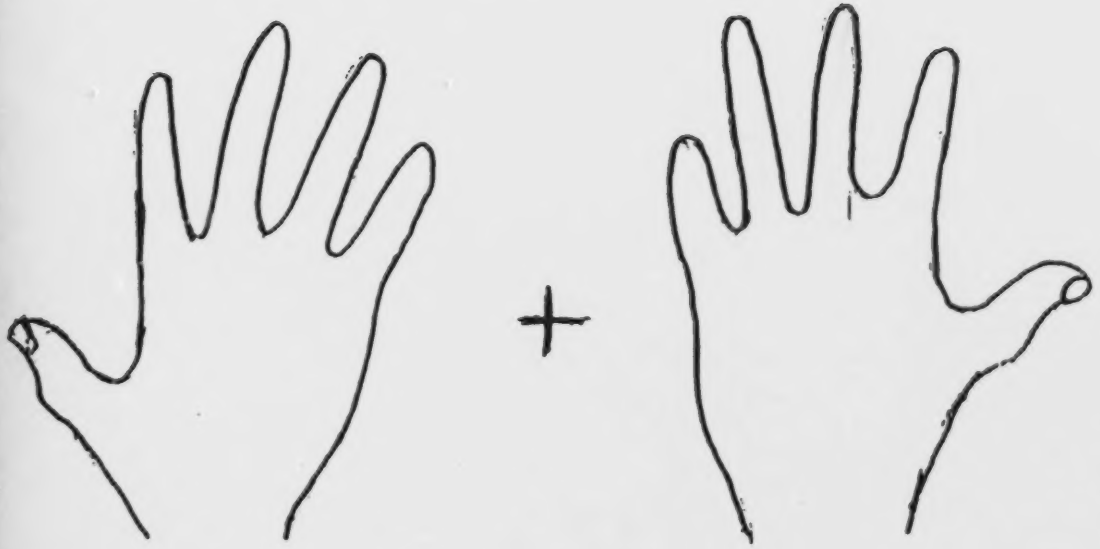
+

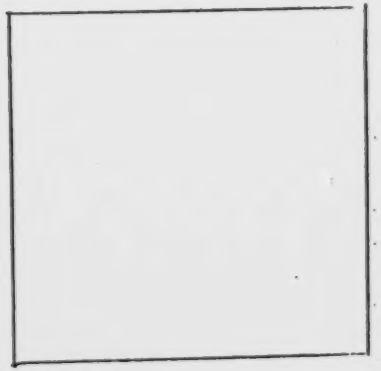
193

194

MATHEMATICIAN:_____

PUT TOGETHER GAME
WORKMAT

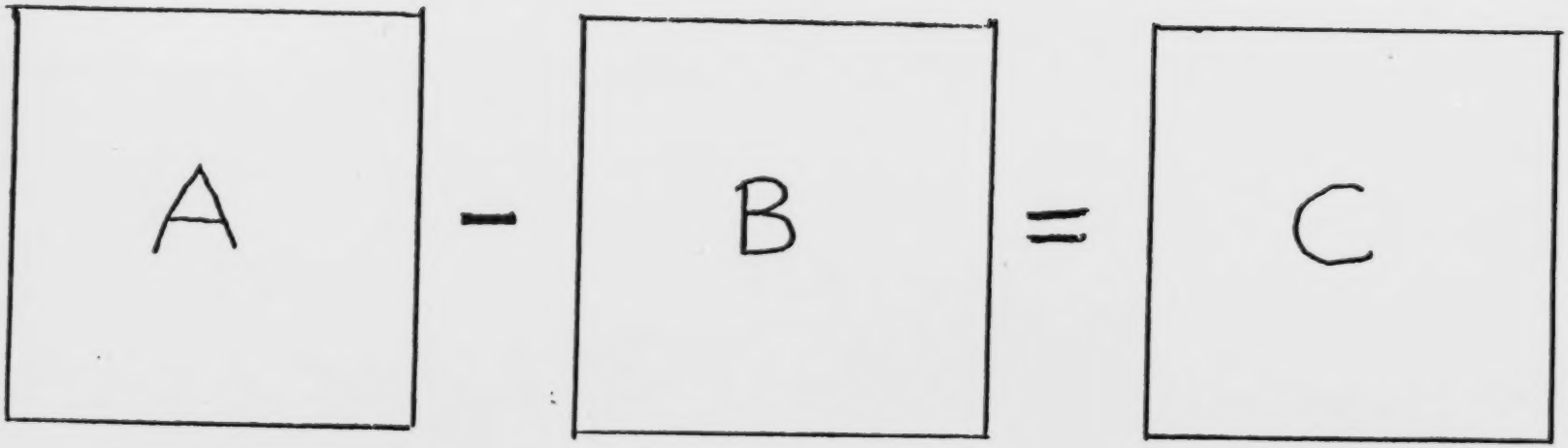




MATHEMATICIAN:_____

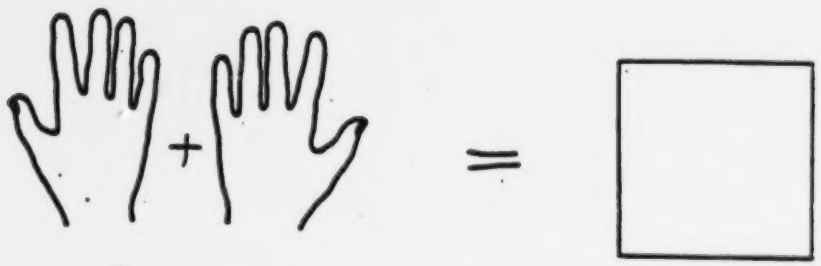
TAKE APART GAME
WORKMAT

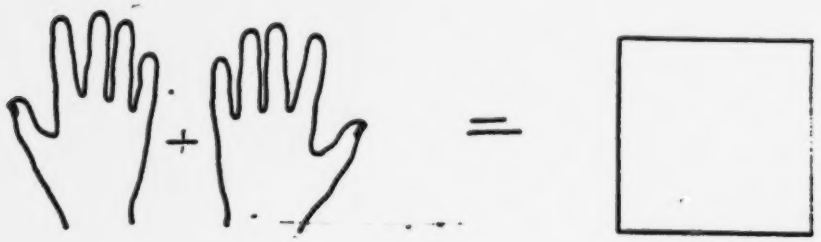
FINDING THE DIFFERENCE GAME
WORKMAT

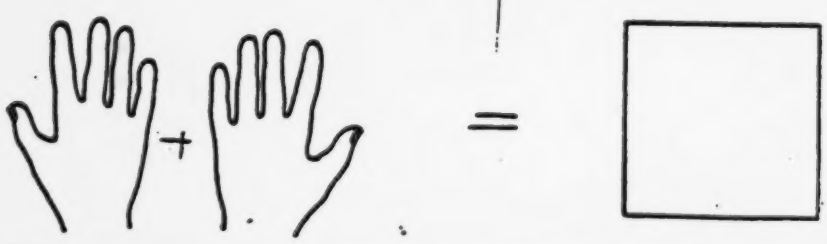


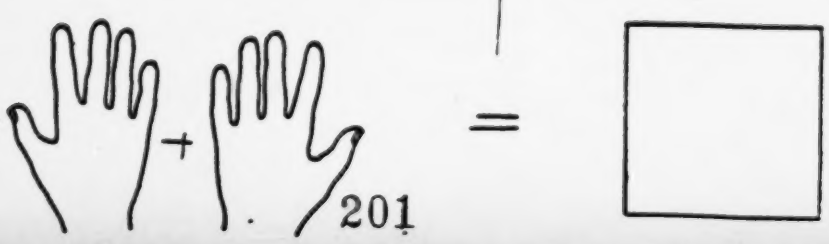
MATHEMATICIAN: _____

THE PUT TOGETHER GAME



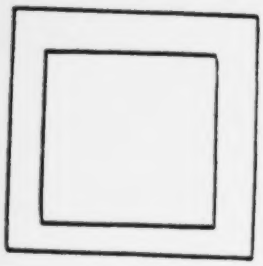
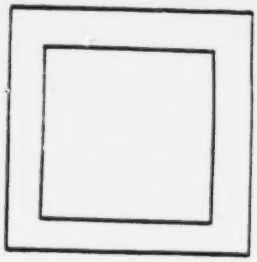
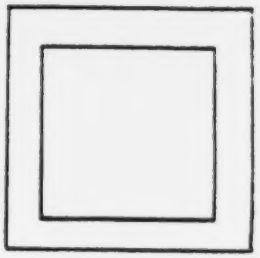
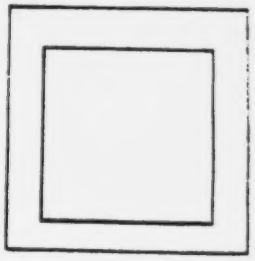




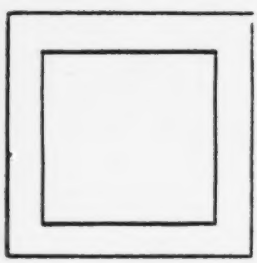
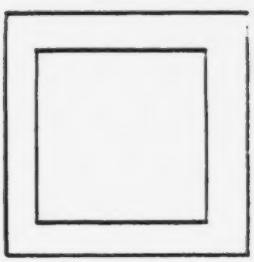
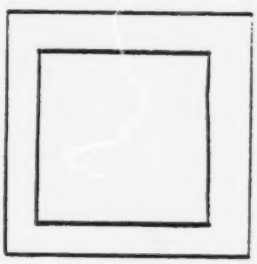
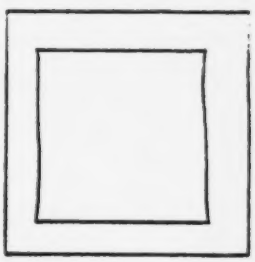


MATHEMATICIAN: _____

My Combine Game Record

+ _____ + _____ + _____ + _____

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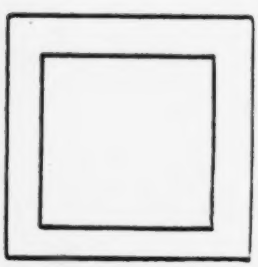
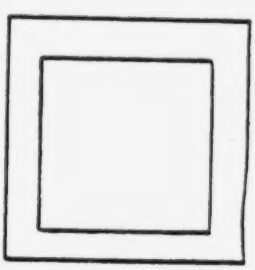
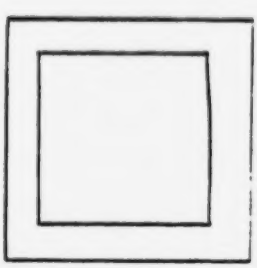
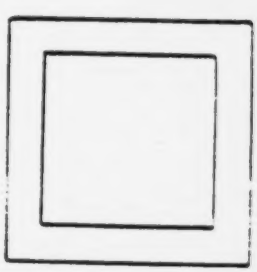
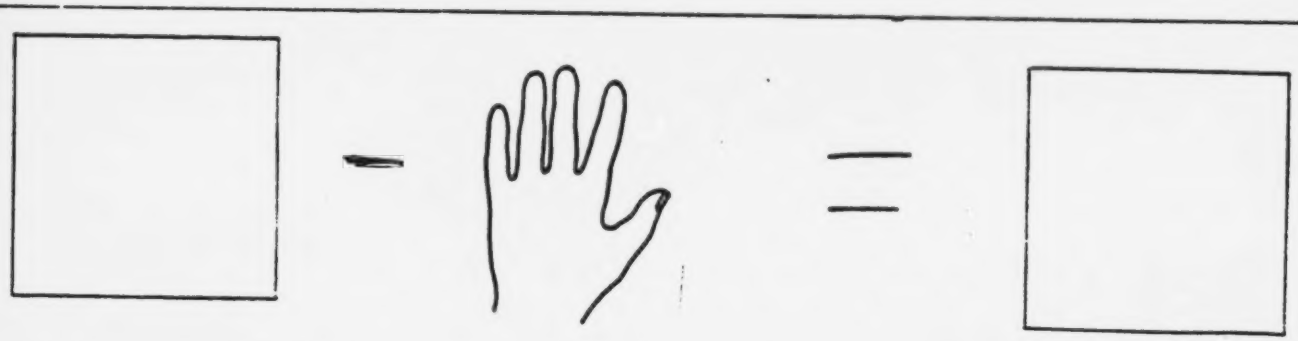





+ _____ + _____ + _____ + _____

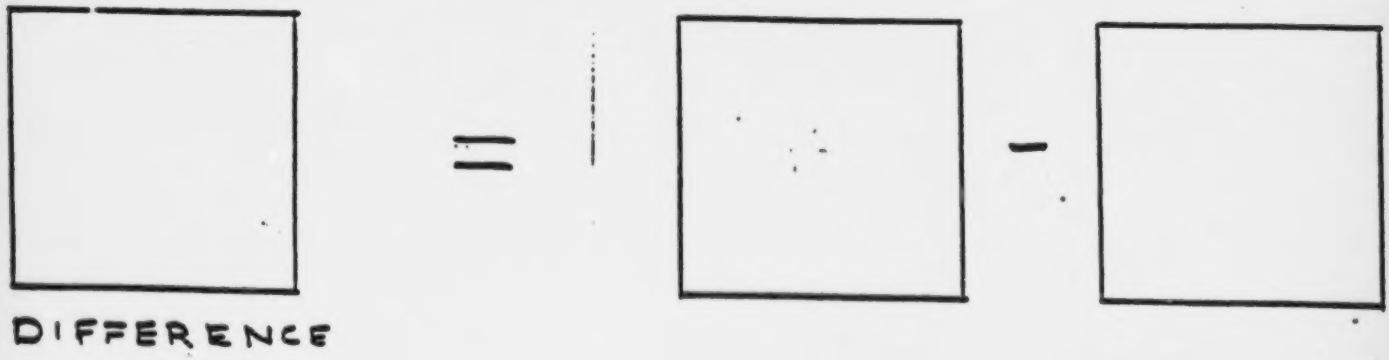
MATHEMATICIAN _____

My Take Apart Game Record

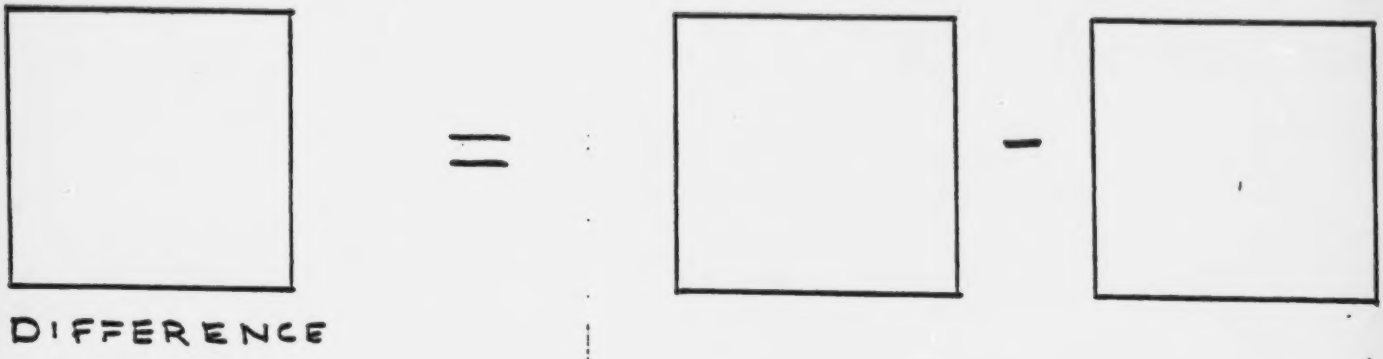


MATHEMATICIAN: _____

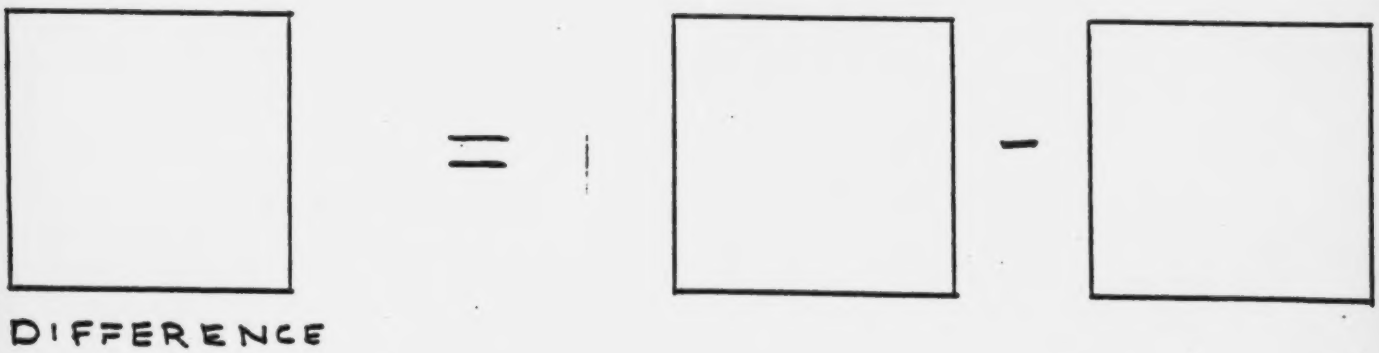
Finding the DIFFERENCE



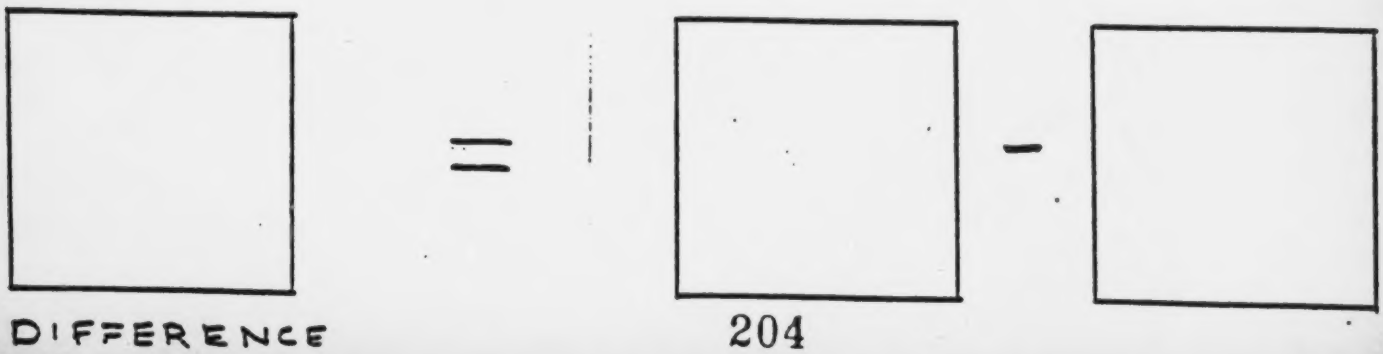
DIFFERENCE



DIFFERENCE



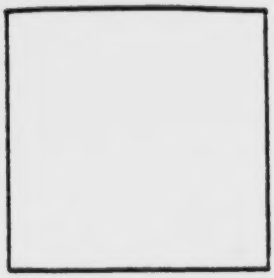
DIFFERENCE



DIFFERENCE

MATHEMATICIAN: _____

My Put Together Game Record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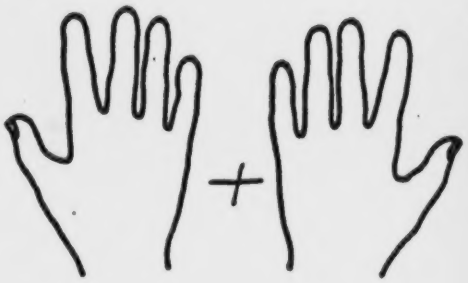

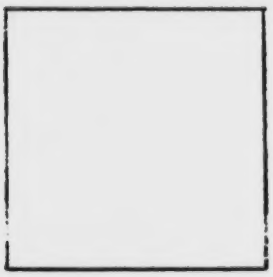
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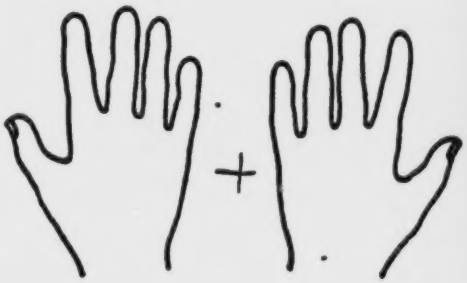

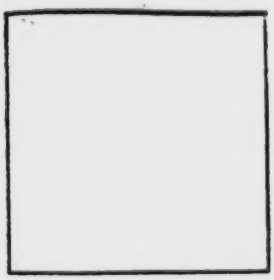
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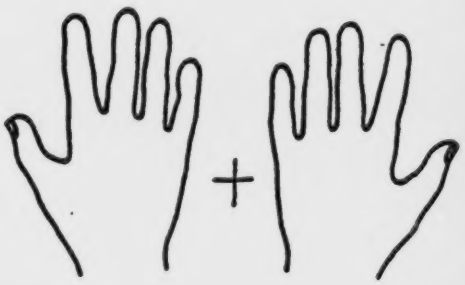

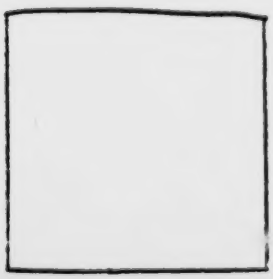
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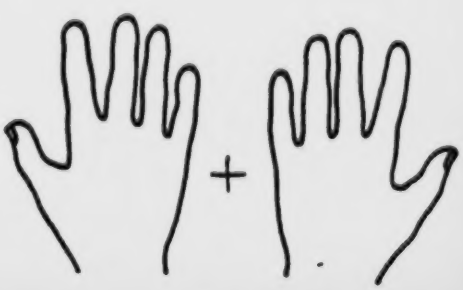
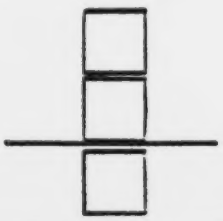
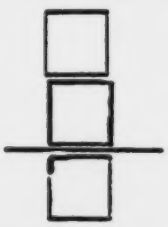
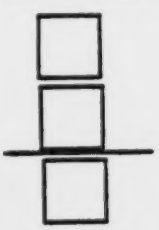
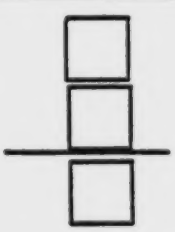
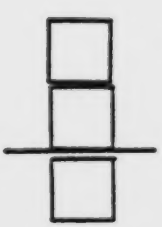


Mathematician: _____

NUMBER SENTENCE	COMPUTATION FORM
	
	
	
	
	

Mathematician: _____

NUMBER SENTENCE	COMPUTATION FORM
□ □ □	
□ □ □	
□ □ □	
□ □ □	

MATHEMATICIAN: _____

NUMBER SENTENCE	COMPUTATION FORM

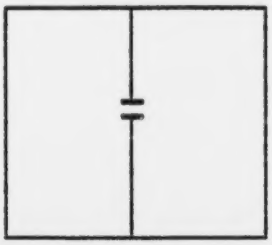
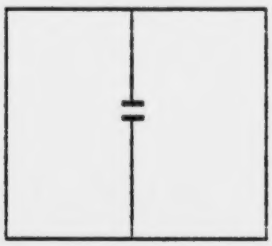
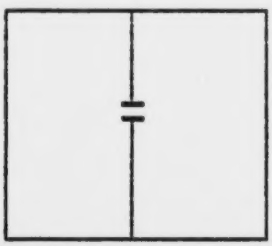
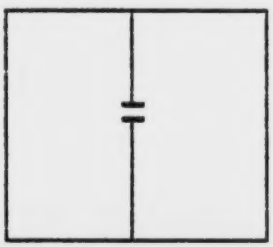
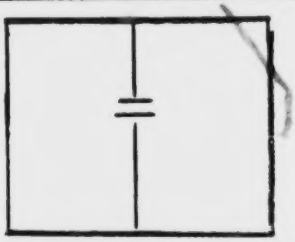
MATHEMATICIAN: _____

MY EQUALITY NUMBER SENTENCES

LEFT SIDE		RIGHT SIDE
	=	
	=	
	=	
	=	
	=	
	=	

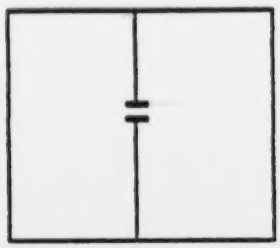
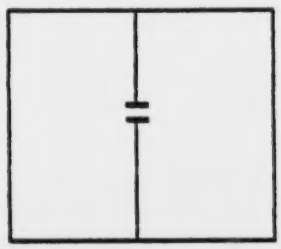
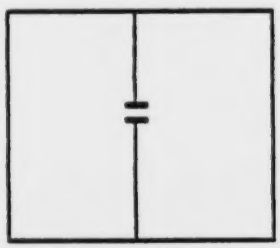
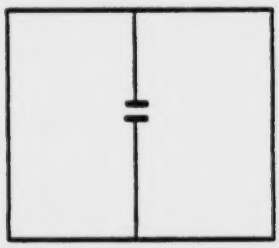
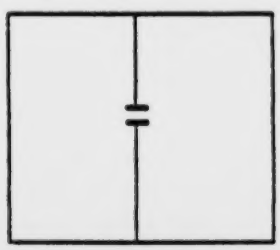
MATHEMATICIAN: _____

EQUALITY RECORD 1A

Equality Board	Number Sentences
	
	
	
	
	

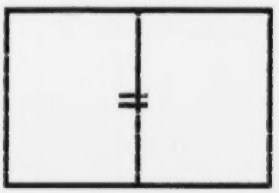
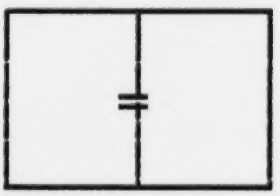
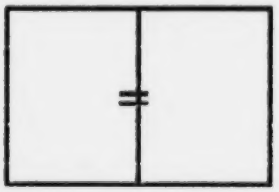
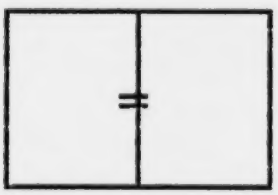
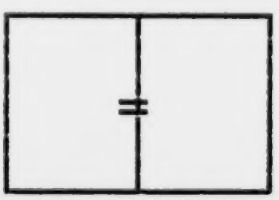
MATHEMATICIAN: _____

EQUALITY RECORD 2

Number Sentence	Equality Board
	
	
	
	
	

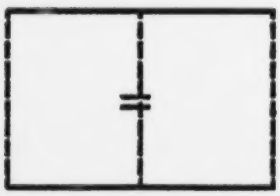
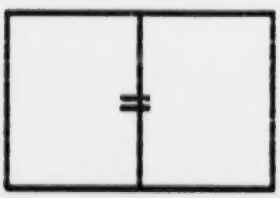
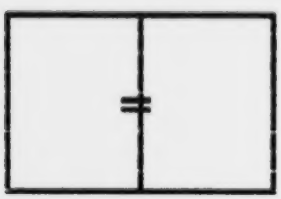
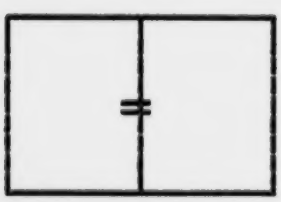
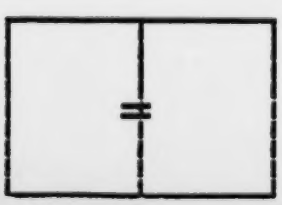
MATHEMATICIAN:_____

MY EQUALITY BOARD RECORDS

NUMBER SENTENCES	EQUALITY BOARD
	
	
	
	
	

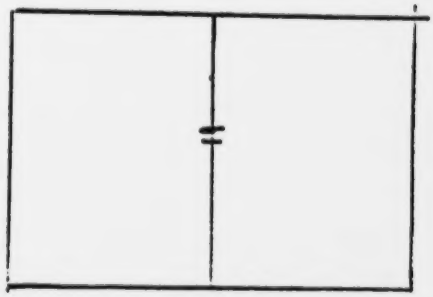
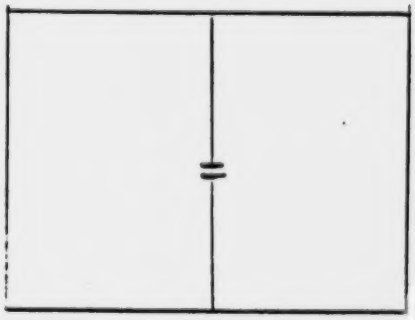
MATHEMATICIAN:_____

MY EQUALITY BOARD RECORDS

EQUALITY BOARD	NUMBER SENTENCES
	
	
	
	
	

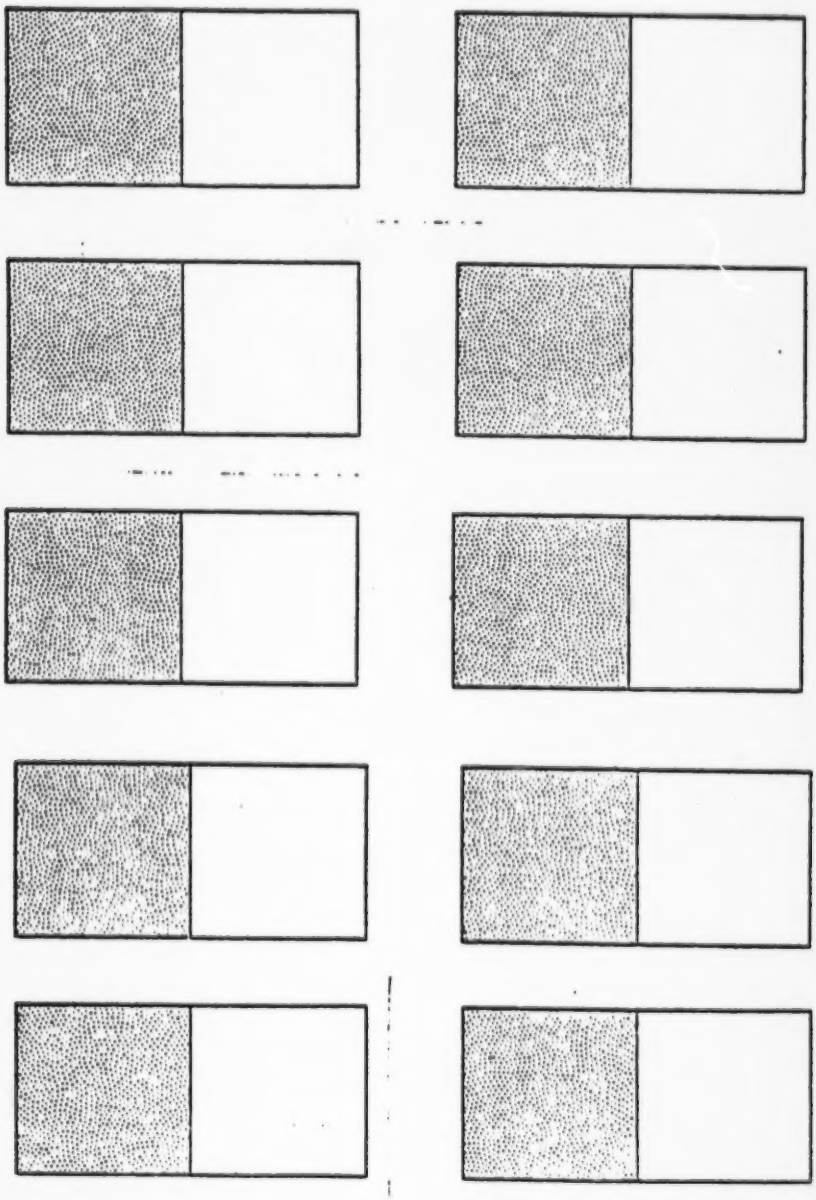
MATHEMATICIAN: _____

The Same Number - Many Ways

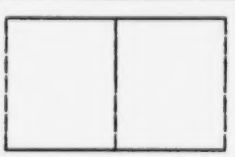
NUMBER	PICTURE	NUMBER SENTENCES
		
		

MATHEMATICIAN:_____

My Counting Game Record



Mathematician: _____

UNIFIX COUNTED	PICTURE OF SPLIT BOARD	NAME IN TENS & ONES
		TENS | ONES
		TENS | ONES
		TENS | ONES
		TENS | ONES
		TENS | ONES
		TENS | ONES
		TENS | ONES
		TENS | ONES

MATHEMATICIAN: _____

Names to Numerals and Back Again

Number Name	Numeral			Numeral			Number Name

MATHEMATICIAN:_____

My Hundreds Computer

Number	Hundreds	Tens	Ones		□	▬	•

Mathematician:_____

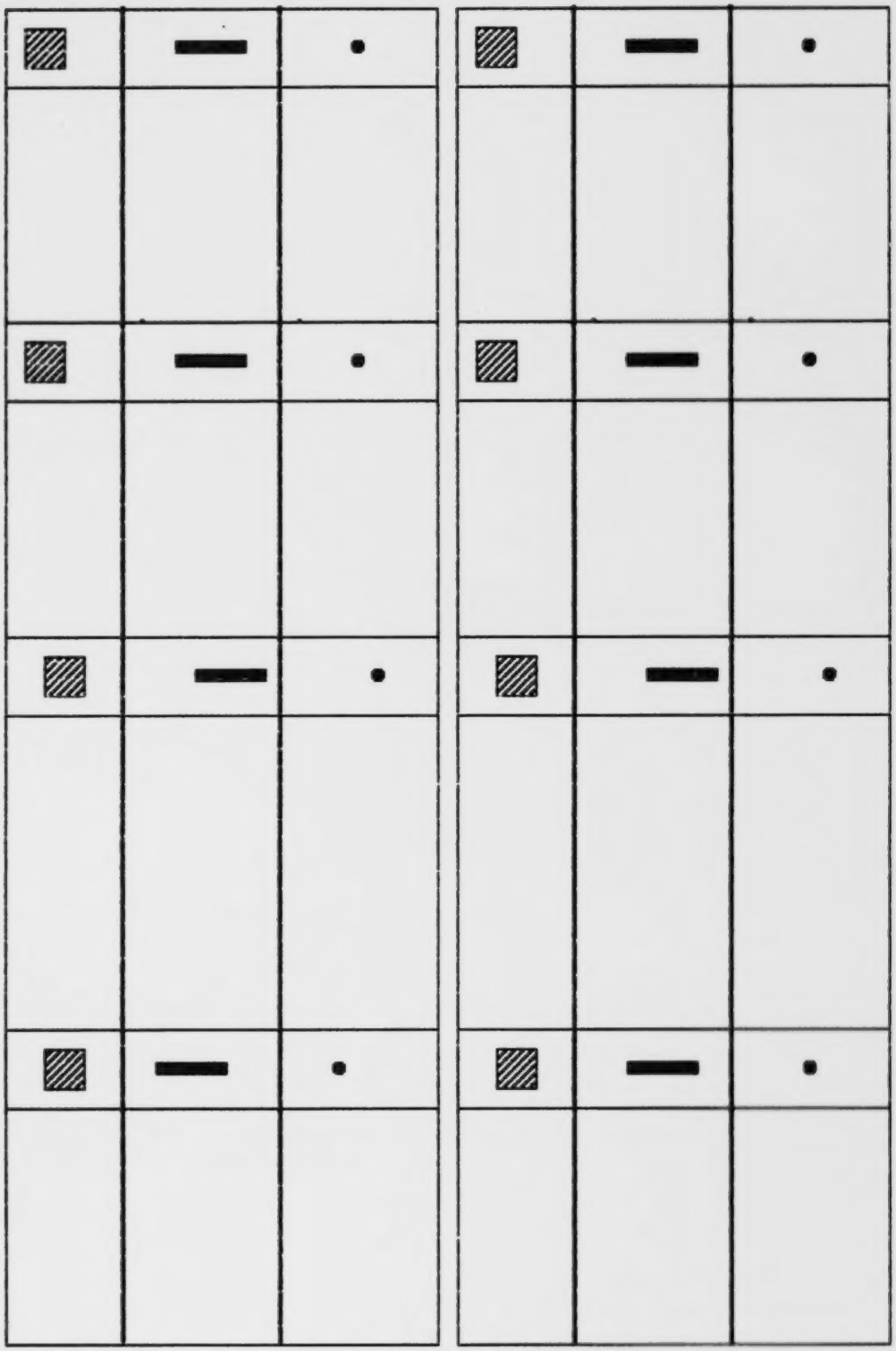
▭	□	TENS	ONES

▭	□	TENS	ONES

MATHEMATICIAN:_____

NAME	PICTURE		NUMERAL	
			TENS	ONES

MATHEMATICIAN:_____ My Work with Numerals



Mathematician:_____ Another Numeral Record

Hundreds	Tens	Ones
Hundreds	Tens	Ones
Hundreds	Tens	Ones
Hundreds	Tens	Ones

Hundreds	Tens	Ones
Hundreds	Tens	Ones
Hundreds	Tens	Ones
Hundreds	Tens	Ones

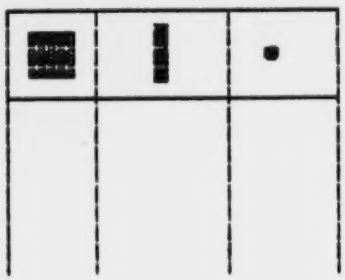
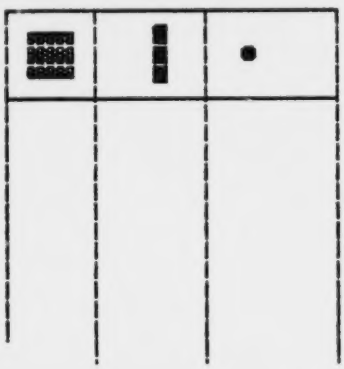
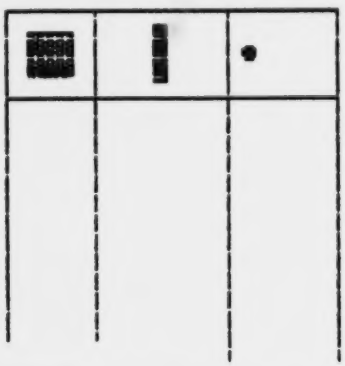
*Mathematician:*_____

Number Words

WORD	PICTURE			NUMERALS		
	▩	|	●	▩	|	●
	▩	|	●	▩	|	●
	▩	|	●	▩	|	●
	▩	|	●	▩	|	●

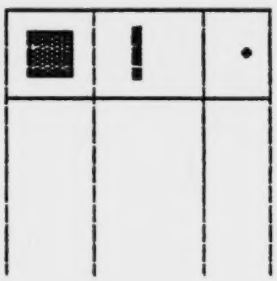
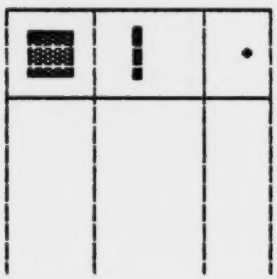
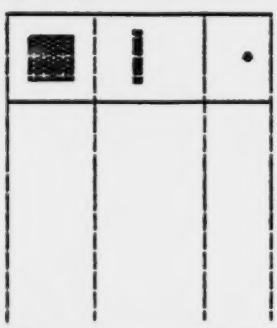
MATHEMATICIAN: _____

WORKSHEET A

PICTURE	NUMERAL	WORD
		
		
		

MATHEMATICIAN: _____

Worksheet B

NUMERAL	PICTURE	WORD
		
		
		

MATHEMATICIAN:_____

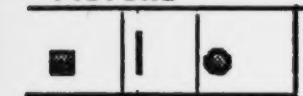
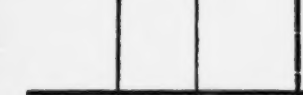
Worksheet C

WORD	PICTURE	NUMERAL
	HUNDREDS | TENS | ONES	
	HUNDREDS | TENS | ONES	
	HUNDREDS | TENS | ONES	

*MATHEMATICIAN:*_____

WORKSHEET D

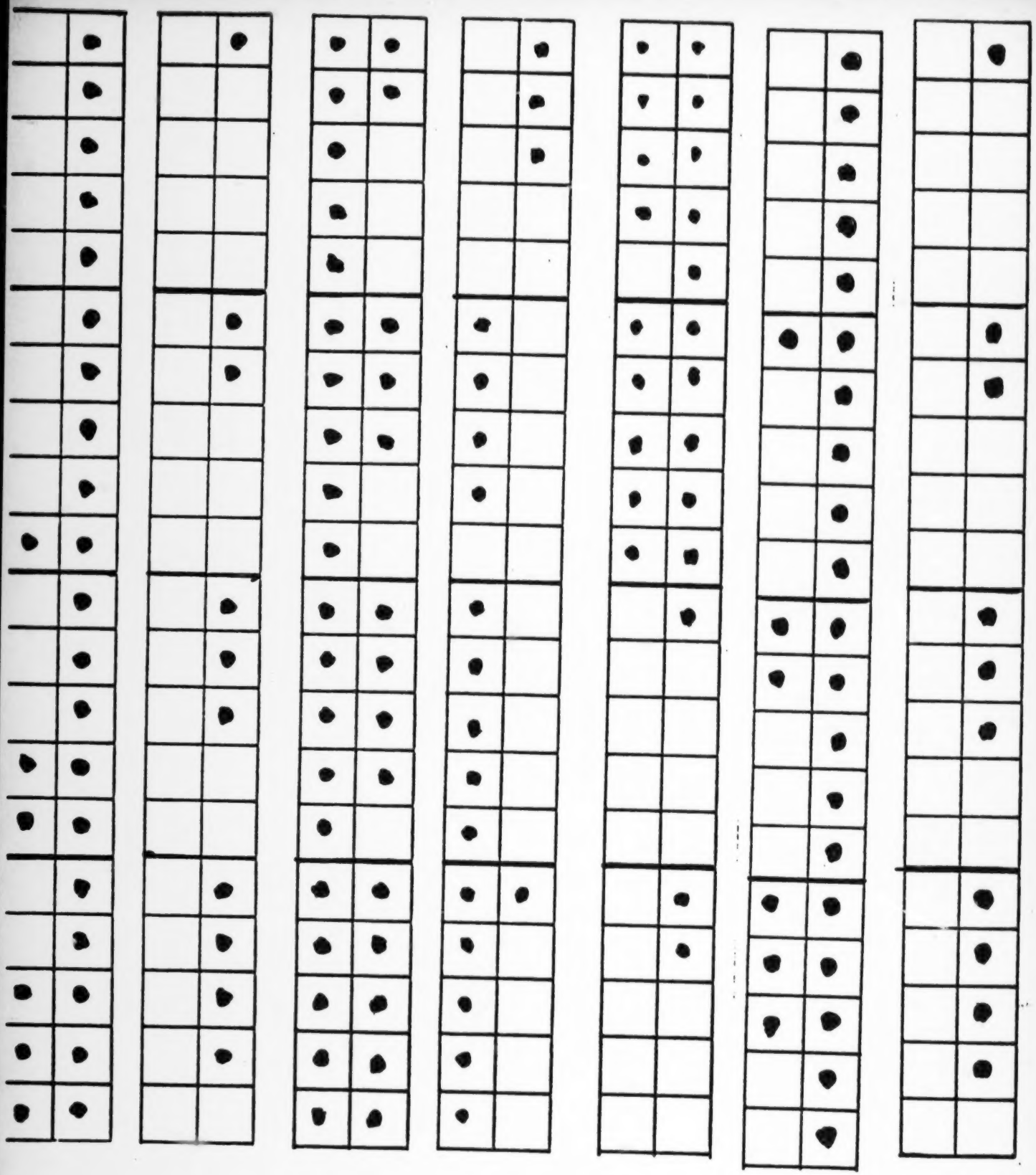
1. Build the number given with base ten blocks
2. Draw the picture, fill in the blanks, or write the numeral as are missing.

PICTURE	NUMBER NAME	NUMERALS
	__Hundreds __Tens __Ones	Hundreds Tens Ones
	__Hundreds __Tens __Ones	Hundreds Tens Ones
	__Hundreds __Tens __Ones	Hundreds Tens Ones
	__Hundreds __Tens __Ones	Hundreds Tens Ones
	__Hundreds __Tens __Ones	Hundreds Tens Ones

*MATHEMATICIAN:*_____

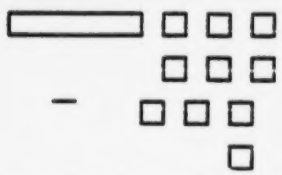
My Record of_____ : _____

No. of Links Used	Color A	Color B	Ratio

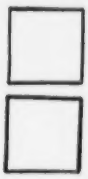
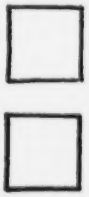


Mathematician: _____

My Base Ten Block Record

Picture Example:	Numerals		Picture	Numerals	
	TENS	ONES		TENS	ONES
	1	6			
	−	4			
	1	2			
	TENS	ONES		TENS	ONES
	TENS	ONES		TENS	ONES
	TENS	ONES		TENS	ONES

Mathematician: _____

TENS	ONES	TENS	ONES
1.			
2.			
3.			

*Mathematician:*_____

My Base Ten Work

Numerals	Pictures		Pictures		Numerals
	TENS	ONES	TENS	ONES	
	TENS	ONES	TENS	ONES	
	TENS	ONES	TENS	ONES	
	TENS	ONES	TENS	ONES	

Mathematician: _____

My Base Ten Block Record

Picture	Numerals		Picture	Numerals	
	▭	□		▭	□
	▭	□		▭	□
	▭	□		▭	□
	▭	□		▭	□

Mathematician: _____

My Base Ten Block Record

Picture Example:	Numerals		Picture	Numerals	
	TENS	ONES		TENS	ONES
	TENS	ONES		TENS	ONES
	TENS	ONES		TENS	ONES
	TENS	ONES		TENS	ONES

*Mathematician:*_____

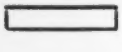
My Base Ten Work

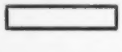
Numerals	Pictures		Pictures		Numerals
	TENS	ONES	TENS	ONES	
	TENS	ONES	TENS	ONES	
	TENS	ONES	TENS	ONES	
	TENS	ONES	TENS	ONES	

MATHEMATICIAN:_____

Picture/Numerals Worksheet

Picture | Numerals

		TENS	ONES

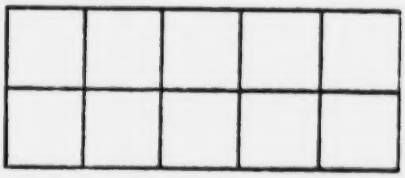
		TENS	ONES

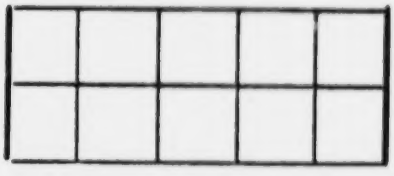
Mathematician: _____

COMPUTATION	PICTURE			NUMBER SENTENCE
	HUNDREDS	TENS	ONES	
+ _____				_____ + _____ = _____
+ _____				_____ + _____ = _____
+ _____				_____ = _____ + _____
+ _____				_____ = _____ + _____
+ _____				_____ + _____ = _____
+ _____				_____ = _____ + _____
+ _____				_____ = _____ + _____

Mathematician: _____

NUMBER SENTENCE	COMPUTATION FORM





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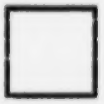
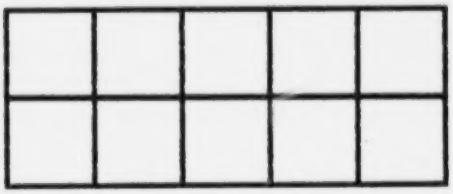
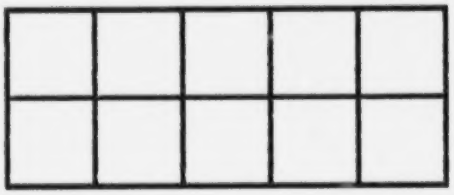
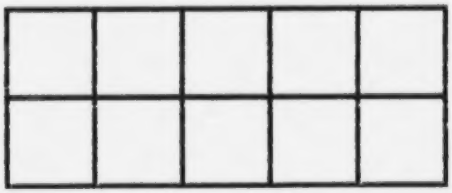
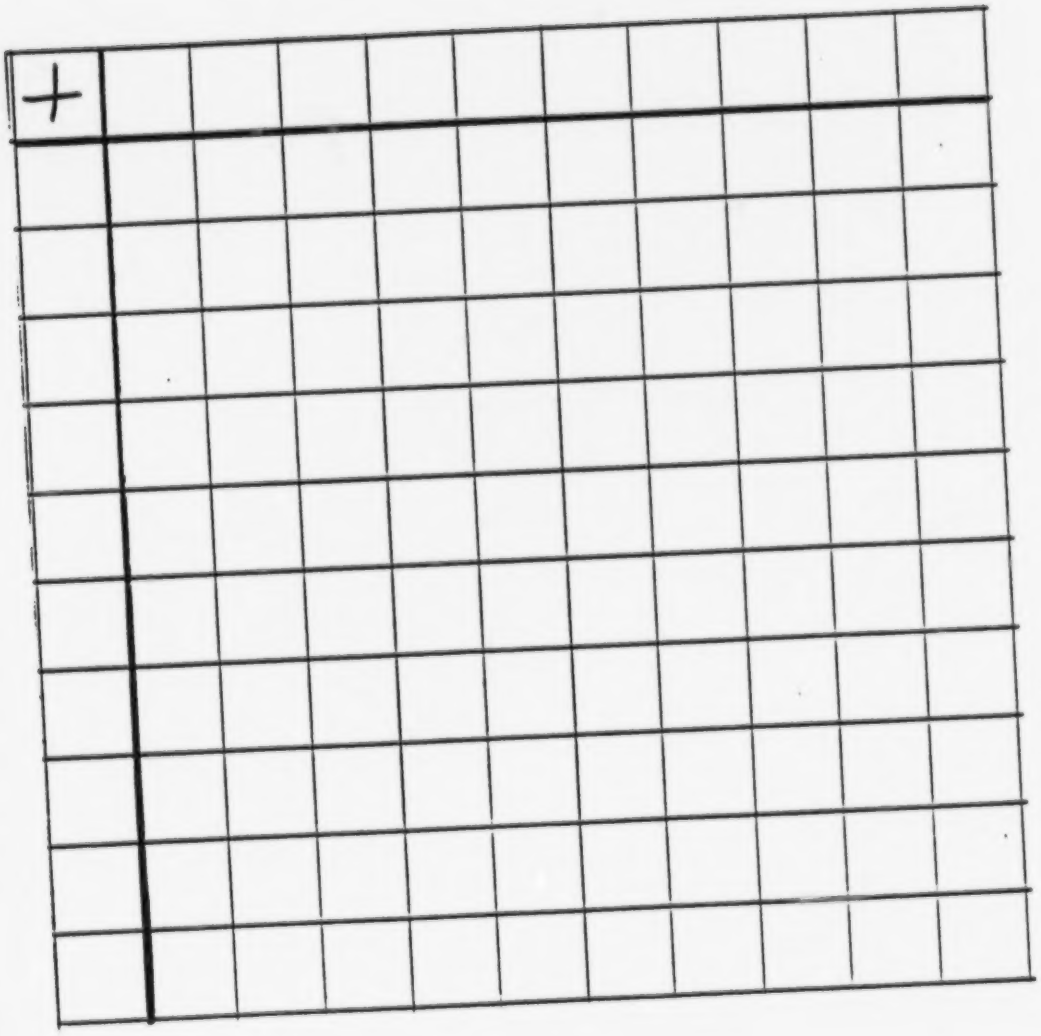


TENS	ONES

TENS	ONES
	 
	

TENS	ONES

HUNDREDS	TENS	ONES



+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

A Sample Worksheet for "Quick Drill" on
"Addition Facts"

These numbers should be randomly arranged

+	3	8	0	5	9	2	4	7	6	1
1										
9										
5										
6										
0										
3										
8										
4										
7										
2										

"HUNDREDS CHART A"

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

"HUNDREDS CHART B"

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Make a template by cutting out those marked with "x"

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Make a template by cutting out those marked with "x"

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

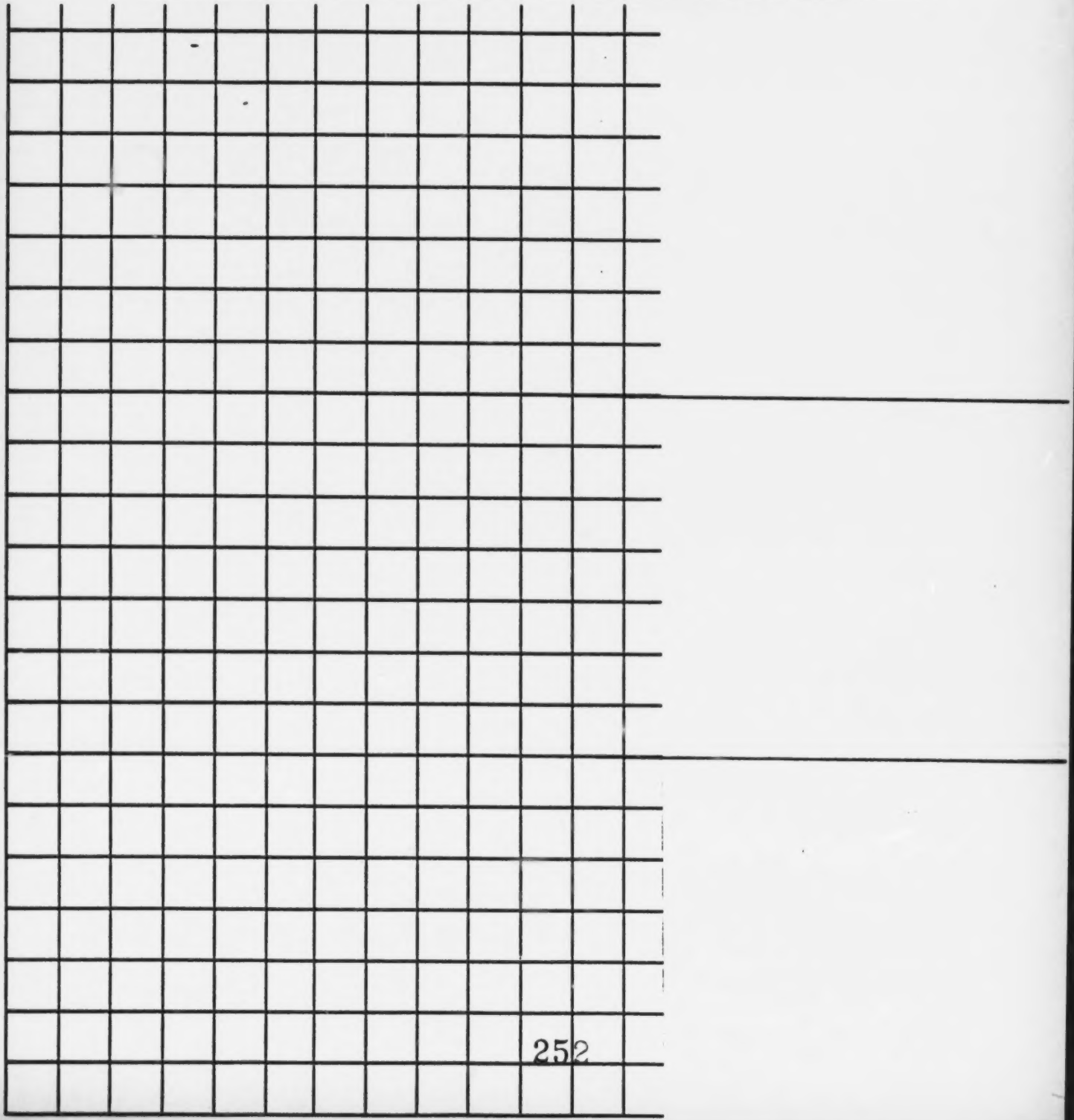
Make template by cutting out those
marked by "x"

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

MATHEMATICIAN_____

Recording Multiplication Rectangles

Tile Rectangle | Number Sentence

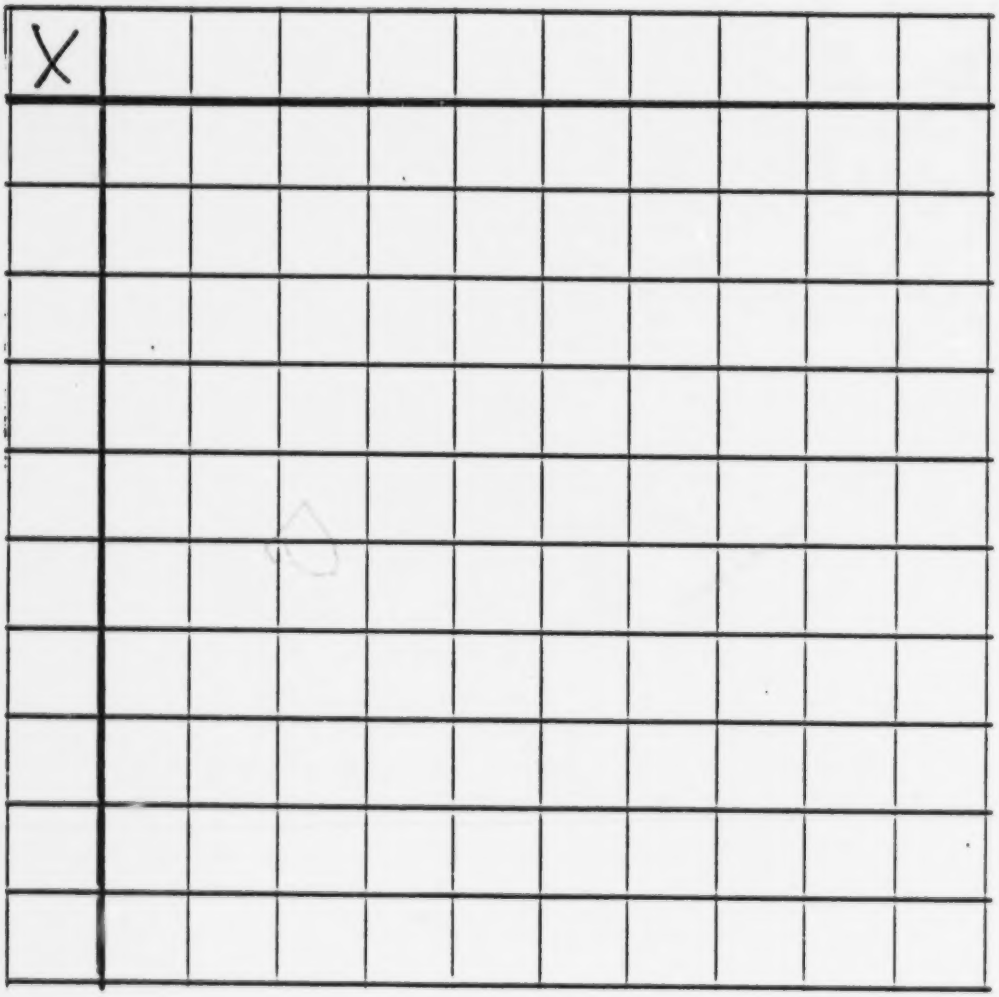


Mathematician: _____

COLUMNS

ROWS

X	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										



A Sample Worksheet for "Quick Drill" on
"Multiplication Facts"

These numbers should be arranged randomly

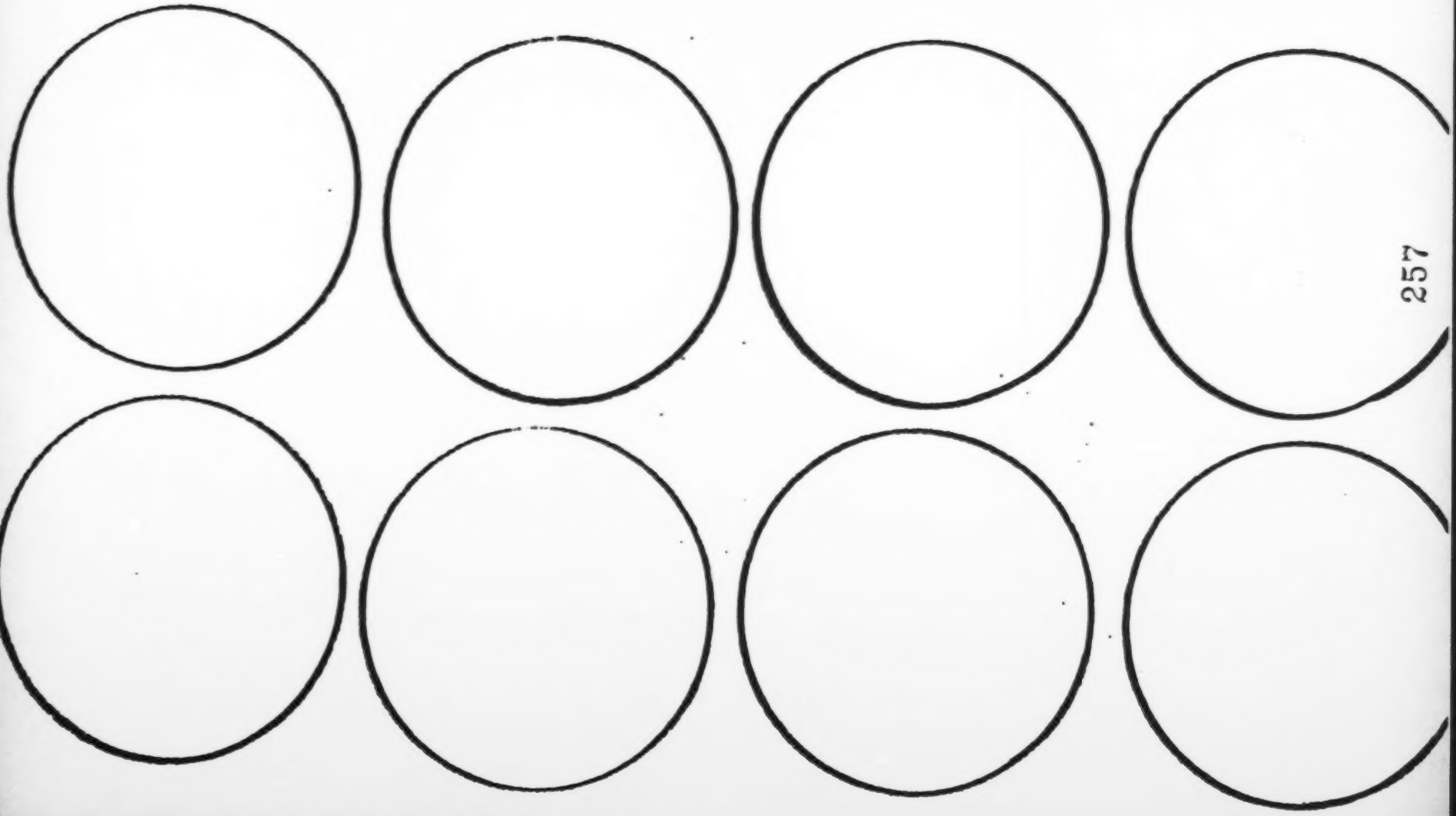
X	2	0	9	5	1	6	4	7	3	8
7										
1										
3										
9										
0										
4										
8										
2										
6										
5										

MATHEMATICIAN: _____

BEANS MULTIPLICATION RECORD

Beans in Each Cup x	Number of Cups =	Beans Used	Number Sentence

BEAM "CUPS"



MATHEMATICIAN: _____

TILE DIVISION RECORD

Tiles Used ÷	Rows	Columns	Remainder	Number Sentence

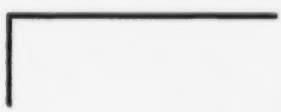
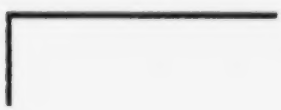
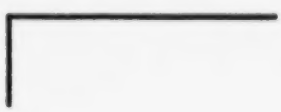
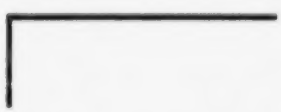
MATHEMATICIAN: _____

TILE DIVISION RECORD

Tiles Used ÷	Rows	Columns	Remainder	Number Sentence

MATHEMATICIAN: _____

A TILE DIVISION RECORD

Tiles Used	Rows	Columns (tiles in each row)	Record
			
			
			
			

MATHEMATICIAN:_____

BEANS DIVISION RECORD 1

Beans Used	÷	Beans in each cup	=	Cups Used	Remainder	Number Sentence

MATHEMATICIAN: _____

BEANS DIVISION RECORD 2

Beans Used ÷	Cups	= Beans needed in each cup	Remainder	Number Sentence

MATHEMATICIAN:_____

Assessment: "Skip Count by (2's, etc.) For Me." Write in the largest number skip counted to for each.

	Skip Counting by:					
	2	3	4	5	10	
Date:						
Date:						
Date:						
Date:						
Date:						

MATHEMATICIAN: _____

Link Size	Number of Links Used	Cubes in the "long" link
Example: 2	2	4
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
3	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
4	2	
	3	
	4	
	5	

MATHEMATICIAN:_____ Pg. 2

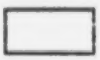
Link Size	Number of Links Used	Cubes in the "long" link
4	6	
	7	
	8	
	9	
	10	
5	2	
	3	
	4	
	5	
	6	
	7	
	8	
6	2	
	3	
	4	
	5	
	6	
	7	
7	2	
	3	
	4	

MATHEMATICIAN:_____ Pg. 3

Link Size	Number of Links Used	Cubes in the "long" link
7	5	
	6	
8	2	
	3	
	4	
	5	
9	2	
	3	
	4	
10	2	
	3	
	4	

MATHEMATICIAN:_____

My Tangram Shapes

Number of Pieces					
ONE					
TWO					
THREE					
FOUR					
FIVE					
SIX					
SEVEN					
EIGHT					

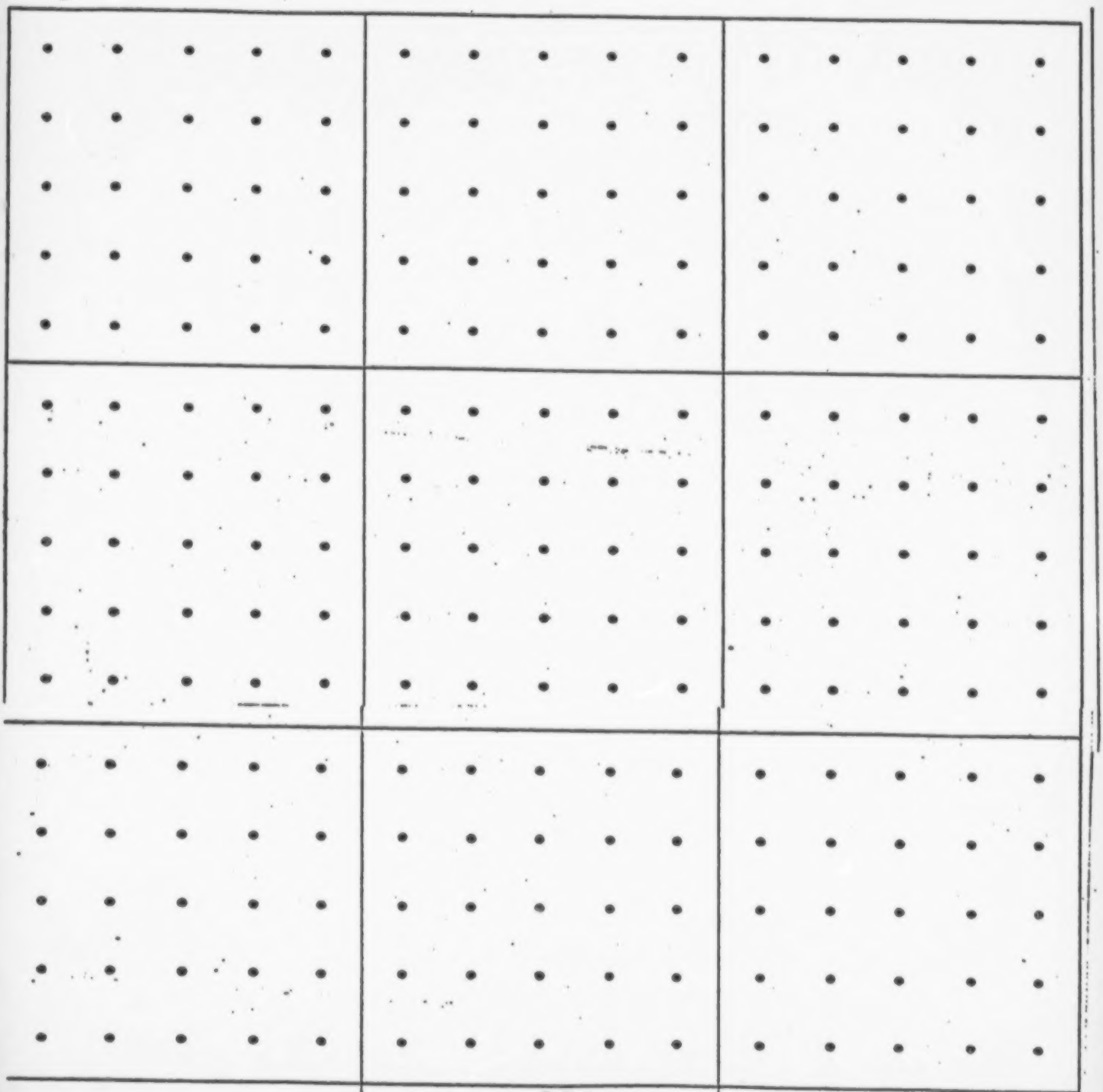
*MATHEMATICIAN:*_____

MY GEOBOARD RECORD

1	2	3	4	5
Number of sides of shapes	Number of pins INSIDE	Number of Pins TOUCHING BAND	Number of Pins OUTSIDE	Number of Pins on the Board

MATHEMATICIAN: _____

My Geoboard Record



PLACE VALUE MAT FOR UNIFIX TEN FRAMES

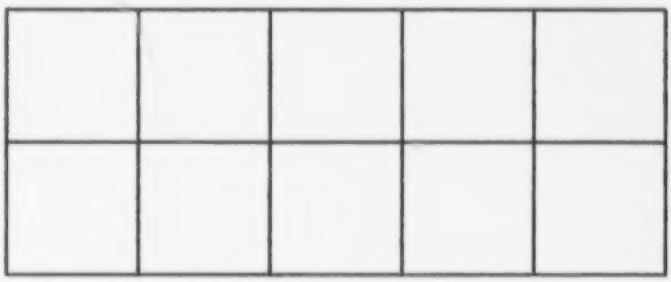
TENS | ONES

3 3/4"

1 1/2"

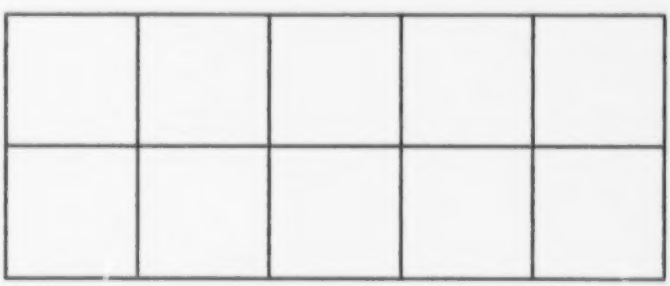
3 3/4"

1 1/2"



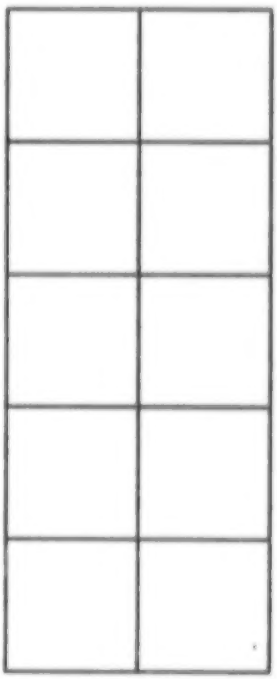
3 3/4"

1 1/2"



PLACE VALUE MAT FOR UNIFIX TEN FRAMES

TENS | ONES



3 3/4"

1 1/2"

TO: THE TEACHER

FROM: A. DEAN HENDRICKSON

Attached is something you can reproduce and send home to parents to encourage them to support what you are doing in the classroom.

MATHEMATICS IN THE HOME

A. Dean Hendrickson, University of Minnesota-Duluth

Introduction:

Mathematics and the use of mathematical thinking is much more than what has been traditional school arithmetic. The arithmetic of whole numbers, fractions and decimals constitutes no more than 10-15% of the mathematics we use throughout our lives. Much of the mathematical reasoning we use can be developed and experienced out of school, particularly in the home. Some of these suggestions may seem remote from the arithmetic you remember, but they will involve children in the THINKING essential to both the learning and use of mathematics in everyday life.

Pre-Mathematical Thinking:

Before a child can understand school mathematics, certain ways of thinking and skills must be available for use. These are continuously used throughout learning of mathematics, but particularly elementary school mathematics. These include: counting, comparing, ordering, using patterns, using grouped material, using language and establishing relations and relationships. Needed experience with these can be obtained around the home. Before describing things to do with children at home to help them with their school mathematics, here are some "golden rules" based upon research and experience with learning children.

1. You must not force children since this has negative effects, such as turning them away from doing things or from you. A child learns when ready, curious, and needing to make sense of something. This goes in spades for drill on memorizing so-called "basic facts."

2. Give children positive things to do when time is available, especially those things they can do and enjoy doing. Don't ask for things beyond the child's capacity to do.

3. Give lots of praise and encouragement. If what the child does or says doesn't seem to make sense to you, don't criticize or correct. Ask questions that might lead the child to consider it in a different way.

4. Don't look for day-to-day progress or change or for immediate results. Just as with many other things, such as walking or talking, a child may seem to be making no headway and then suddenly, it's all there. Children develop in spurts and unevenly, and have long plateaus where nothing seems to be happening. That's normal and accept it. There is probably a lot going on below the surface.

5. Don't compare yours with other children. Everyone is different - thank goodness!

6. Don't worry if a particular skill, such as using language, is

coming along more slowly than you'd like or than brother John's did. Somehow most of them seem quite a lot alike by the time they are 12 or so.

Words:

A number of words commonly used in mathematics and related to teaching mathematics should be used often outside of school as well. Some examples are *some-more, a lot, more than, less than, large, small, many, few, same as, different, alike, all, some, not, left, right, ahead of, behind, above, below, front, back, long, short.*

In addition to words associated with comparing, grouping and space, the number words are important. Children must know the counting words, but even more than that, they must see the pattern in the use of counting words. The cardial words like *first, second, third,* et. are also important. Use of these words around home helps children to count objects correctly and to identify position of things in ordered arrangements.

Comparing:

Have children compare things as to size, length, are and volume whenever possible. "Which glass has more?" "Which box holds more?" "which of these is heavier? heaviest?" "Put these sticks in order of length." "Arrange the silverware so the longest is farthest from the plate and the shortest is nearest the plate." Questions like these should be frequent. They should involve different kinds of things both indoors and outdoors. Combine these with questions that make the children estimate measurements of distance and height such as "Which do you think is as high as the shed, A or B?"

Comparing of quantity leads to better understanding of number and number relationships. "Are there more chairs or lamps in this room?" "Are there more cups or teaspoons on the table?" "Have we go more red roofs or green roofs on our street?" "Put enough table knives on the table so that there are as many knives as forks." "Do you have more boys or girls in your class?" These can be asked when out walking, riding in the car, watching TV or sitting in the boat. Ask children to do things that will make one group as large as another frequently All such activity helps children build number relations into their deeper understandings, instead of as memorized associations that have no meaning - like names and dates you once memorized to pass a history test!

Ordering things that can be counted is important. Bead stringing activities are good for young children. "String some beads so the third bead is red and the fourth bead is blue." "Make a string so every other one is green," etc.

Ordering things that have lengths, areas and volumes extends comparing beyond two things. Have children place three sticks of different lengths in order from shortest to longest; place three pieces of paper of different areas into orders; place three different sized cans of jars into order. Gradually extend the number of things to more than three for these activities.

Ask frequent questions about the ordering of events as to which happens first, second...last, etc. Connect these with time estimations, "How many minutes ago do you think this happened? How many days?" etc.

Counting:

Children should keep extending their memorized sequence of counting words. This is important. But being able to say the words in right order does not mean they can count things. They need much practice at this. Have them count everything around the house that is countable - the chairs, tables, legs on chairs; the tiles on the floor, in the ceiling; the number of windows in a room; the silverware in the drawer; the cans on the shelf; the pieces of wood in the woodpile; the telephone poles going by, etc. The more they count, the better able they are to count. When they are pretty good at counting forward, have them do some counting back. For example, start with 20 clothespins. One at at time put one into a can and count aloud those that are left as each one is removed from the pile.

Patterns:

Have children look for patterns - in the carpet, in the ceiling, in wallpaper, in the drapes, on the bedspreads. Patterns of shape, or color, or sound are all important. Beads can be strung in patterns. Collections of bottle caps, old keys, buttons, screws, nuts and bolts, and similar "junk" can be put into patterns. Ask children what would come next in a pattern, or what would go where something is missing in a pattern.

Number:

Help your child learn number size by having him see the same number, such as five, in many different arrangements and materials. Playing cards can be sorted into those all having the same number. Mixed groups of say, five marbles, three buttons, three keys, six spoons, can be used. "Find me the material there are five of," etc. Put some number, seven for example, of beads or marbles into three or four different shaped glass jars, "Find a jar with seven in it." "Find another." Put the same number of one kind of thing in one jar and another kind in a second jar, etc., and do the same kind of thing. Involve the child with numbers in as many different ways, with as many different kinds of material, and as many different sizes as possible. Gradually increase the number size as the child seems able to easily handle smaller numbers.

Using Numbers:

Comparing groups with number property; combining such groups; separating larger groups into smaller groups of a given size or into equal size groups - all of these activities help children to understand when each of the four arithmetic operations are used.

Some examples of things to do in the home of this kind are:

1. Compare two different sized groups in several ways. "How many more are there in this group than in that group?" "This group has how many fewer than that group?" "How many times as many are there here as there?" These kinds of questions used with groups of all kinds

of things - knives, forks, chairs, chair legs and table legs, buttons, marbles, pieces of candy, etc., help the child with what the school is doing.

2. Join together several groups of the same size into a larger group. Rows of pennies can be arranged into an array like this and can then be looked at a different way to see 5 groups of 6 pennies:

Both lead to a total of 30 in the array. Do this in a row at a time, having the child tell you how many are there all together each time. Separate and take apart such arrays row by row and see what is left each time. Do this with different kinds of things, different size rows and different total numbers of things. Clothes pins, ceramic tiles, beans, corn are all good for this.

3. Join together groups of different size, such as seven things with five things. Have the child describe what is happening in words. Have the child add to one group of things enough to make it the same size as another larger group. Have the child make equal two unequal size groups without adding anything more to the collection. "Here are a group of 15 clothes pins and one of 7 clothes pins. Do something so you have two equal groups."

4. Give the child large amounts - in the 20's or 30's of things to:

   a) make several groups of a given size from. Some numbers should make these smaller groups an even number of times and some should have some left that is not enough to make another of the smaller group.

   b) make a certain number of groups that will all have just as many in them.

   Examples:

   "Put these 30 beans into 6 cups, so each cup has just as many. How many are in each cu?"

   "Put these 43 beans, six at a time into cups. How many cups did you use?" "What should e done with what is left over?" "When do you have some left over?" "When don't you have anything left over?"

When you do for walks, have the children compare, add together, etc., things along the way. Do the same in the car, the supermarket, in the drugstore. "How many are there on the top shelf?" "How many re on the bottom shelf?" "How many are there on the top and bottom shelves together?"

Have the child do as much adding, subtracting, multiplying and dividing of this kind - always as related to things - as you can. <u>DON'T</u> try to drill your child on "addition" facts or "multiplication" facts. Let the child learn these in due time

through the school activities and those you do at home as described here. DON'T have the child write number things - the school will do this. Accept verbal answers and descriptions. Get in the habit of asking your child why certain answers are given and LISTEN.

SOME FINAL HINTS:

1. Have your children count things as much as possible.

2. Ask children simple addition, subtraction questions about REAL things in the surroundings to give practice in mental arithmetic.

3. Play card games that require mathematics or related things like WAR, OLD MAID, CRIBBAGE, RUMMY (regular or gin).

4. Give thinking games for holiday gifts - CONCENTRATION, HUSKER DU, etc.

5. Get a Little Professor or some similar calculator-based program to give mental arithmetic practice.

6. Cheap mathematics games can be bought at Target, Woolworths, etc. Some examples are COVER UP, HEADS UP, SCORE FOUR, TUF, APOLLO, etc.

7. Give your child a simple four function calculator and let him or her fool around with it.

8. Encourage block play and building, sand play, making birdhouses, etc.

9. Key words are COMPARING, COUNTING, PATTERNS, COMBINING (groups), SEPARATING (large groups into smaller groups)

10. Point out mathematics wherever it is in the surroundings. Children must realize mathematics is:

    a. easy to learn
    b. useful
    c. fun

LEVEL TWO
INTRODUCTION

Coming into this level students will have had work with:

1. Number concept development
2. Meaning and use of the four arithmetic operations
3. A basis for place value representation
4. Problem solving
5. Recognition of 3D and 2D geometric shapes
6. Logic
7. Graphing
8. Measurement

All of this has been introductory, and, in order to continue with the children, you must do some assessment of this development prior to starting.

While emphasis is on mastery of each concept, it is a good idea to periodically build in an activity that reviews all concepts and skills previously learned. Once each week is a good schedule for elementary school. This can be done orally in the primary grades and with a written exercise after symbolic representations have been mastered. Since all operations arise from combining, separating, comparing and part whole relations, as soon as the operation symbols are understood, these operations should all be presented at the same time in the written review exercises.

It is easier to do this at the beginning levels with orally presented review activities.

It is axiomatic for good teaching that past learnings should ALWAYS be integrated into new learning activities.